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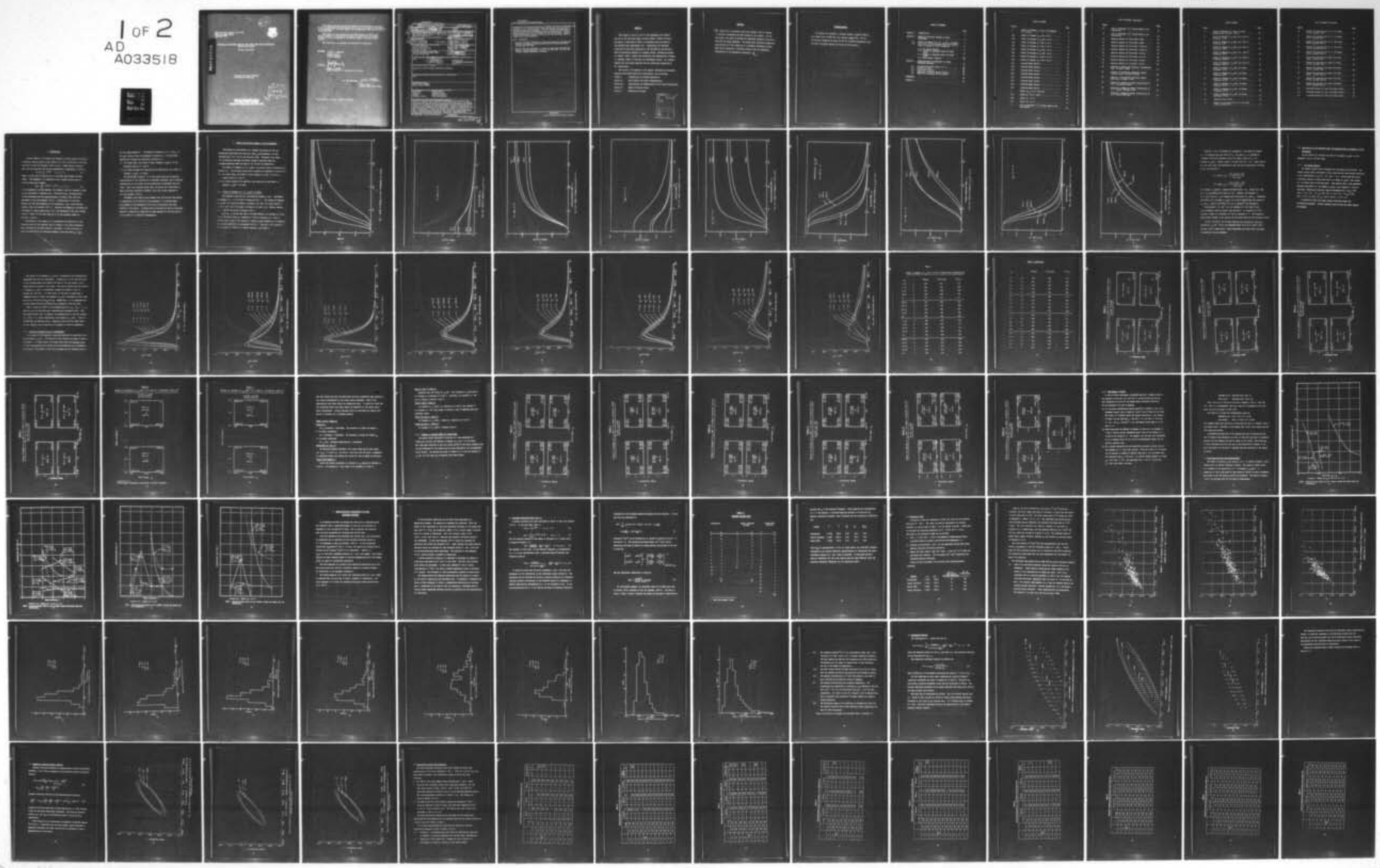

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RADC-TR-76-294, Volume V (of five)
Final Technical Report
September 1976

RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBUTION
Sensitivity Analyses

Syracuse University

ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report provides all the necessary information to make practical application of acceptance tests based on a prior distribution, when an exponential distribution of failures and an inverted gamma prior distribution are assumed. Volume I, "Introduction and Problem Definition," presents an introduction to the problem and a summary of the other volumes. Volume II, "Risk Criteria and Their Interpretation," defines the different statistical risks which can be used to characterize the tests and provide a guide for selecting appropriate risks in various practical situations. Volume III, "Implications and

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Determination of the Prior Distribution," provides the means for determining the parameters of the prior distribution from existing data, and discusses the reason for using an inverted gamma. Volume IV, "Design of Testing Plans," provides instructions for establishing a test time and number of allowable failures based on the prior distribution and the selected risks. Volume V, "Sensitivity Analyses," shows the effects on the test parameters caused by changes in the prior parameters.

Block 7 (Continued)

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PREFACE

This report is one of a set of five presenting the results of part of the work done under contract number F-30602-71-C-0312. The report is delivered to RADC in accordance with item A006 of the Contract Data Requirement List. Sponsorship and technical direction of this task originated in the Reliability and Maintainability Engineering Section (A. Coppola, Chief), Reliability Branch (D. Barber, Chief), within the Reliability and Compatibility Division (J. Naresky, Chief) of the Rome Air Development Center. Mr. Anthony Coppola was the Project Engineer who was technically supported by Mr. Jerome Klion.

The titles of the reports on the subject "Reliability Acceptance Sampling Plans Based Upon Prior Distribution" are as follows:

Volume I.	Introduction and Problem Definition.
Volume II.	Risk Criteria and Their Interpretation.
Volume III.	Implications and Determination of the Prior Distribution.
Volume IV.	Design of Testing Plans.
Volume V.	Sensitivity Analyses.

ABSTRACT

Sensitivity of designed plans and computed risks to changes in the prior parameters has been studied in this report. In the first part, the study is pursued for specified changes in the prior mean and the shape parameter. The second part involves an empirical quantification of the uncertainty in parameter estimates by using Monte Carlo simulation, likelihood contours and the asymptotic distribution of the parametric estimators.

ACKNOWLEDGMENTS

The authors are thankful to Anthony Coppola, Anthony Feduccia and Jerome Klion of RADC for their helpful suggestions. We are particularly indebted to Donald Wu for his valuable programming help and other assistance during the course of this project.

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1. INTRODUCTION

Various aspects of the design and analysis of single sample reliability acceptance sampling plans in the presence of a prior distribution have been described in Goel and Joglekar (1976 a,b,c,d). These reports primarily deal with the case when the failure distribution is exponential, given by

$$f(t|\theta) = \frac{1}{\theta} \cdot e^{-t/\theta} \qquad t \geqslant 0,\ \theta > 0 \tag{1}$$

where t is the time to failure and θ is the mean time between failures (MTBF). The parameter θ is construed to be a random variable with an inverted gamma prior density

$$g(\theta) = \frac{\gamma^{\lambda}}{\Gamma\lambda} \cdot \theta^{-(\lambda+1)} \quad e^{-\gamma/\theta},\ \lambda,\ \gamma,\ \theta > 0. \tag{2}$$

In the presence of a prior density, the producer's and the consumer's risks can be quantified in different ways. The definitions, interpretations, interrelationships and the appropriateness of various risks have been discussed in Goel and Joglekar (1976 b). Determination of the prior density $g(\theta)$ and the estimation of the parameters γ and λ have been described in Goel and Joglekar (1976 c). Numerical and graphical procedures for the design of single sample plans (T,r^*) are discussed in Goel and Joglekar (1976 d), where T is the test time and r^* is the allowable number of failures in time T.

The purpose of this report is to investigate the sensitivity of the designed plans and the computed risks to changes in the prior parameters. This investigation has been pursued in two parts. In the first part we study the sensitivity for specified changes in the prior mean $\mu_p\left(= \frac{\gamma}{\lambda-1}\right)$,

and the shape parameter λ. The effect of changes in T^* ($= T/\theta_0$), r^*, and μ_p on various risks is considered in Section 2.1. The following sensitivity analyses are performed in Section 2.2:

(i) For fixed risks, the effect of small changes in μ_p and λ on the designed values of T and r^*.

(ii) For plans designed with specified risk combinations, the effect of changes in μ_p and λ on risks.

The second part (Section 3) of this study involves an empirical quantification of the uncertainty in parameter estimates, and a numerical determination of the effect of such uncertainty on designed plans and risks. Since this requires actual data, we pursue this study based on some of the data reported in Schafer (1970) and further analyzed in Goel and Joglekar (1976 c).

Throughout this study we have assumed that the failure distribution is exponential and the prior for the parameter θ is inverted gamma. Only changes in the parameters of the prior distribution have been studied in this report. It should also be pointed out that an analytical approach to sensitivity study did not seem possible in this case and we had to resort to a numerical investigation.

2. EFFECT OF SPECIFIED CHANGES IN PRIOR PARAMETERS

The purpose of this section is to examine the effects of the uncertainties associated with the prior mean μ_p and parameter λ on the designed plan (T,r^*) and on the selected risks. Throughout this study, for illustration purposes the design values of specified MTBF and minimum acceptable MTBF are taken to be 100 and 50 respectively.

The effect of changes in T, r^*, μ_p and λ on various risks is considered in Section 2.1. The following sensitivity analyses are performed in Section 2.2.

(i) For fixed risks, the effect of small changes in μ_p and λ on the designed values of T and r^*.

(ii) For plans designed with specified risk combination, the effect of changes in μ_p and λ on risks.

2.1. Effect of Changes in T, r^*, μ_p and λ on Risks

The classical risks are not influenced by changes in $g(\theta)$. The effect of changes in T, r^* on $\bar{\alpha}$ and $\bar{\beta}$ is shown in Fig. 1. The effects of changes in μ_p and λ on $\bar{\alpha}$ and $\bar{\beta}$ are shown in Figures 2 (a) and 2 (b) while those of T and r^* and $\bar{\alpha}$ and $\bar{\beta}$ are shown in Figures 3 (a) and 3 (b). Similar results for α^* and β^* are given in Figures 4 and 5.

From Fig. 1, we see that when r^* is kept constant, an increase in T from 100 to 200 hours leads to a decrease in β and an increase in α. This is obvious since larger T for constant r^* implies larger probability of rejection and smaller probability of acceptance for all θ. Similarly, for constant T, an increase in r^* from 2 to 4 implies smaller α and larger β.

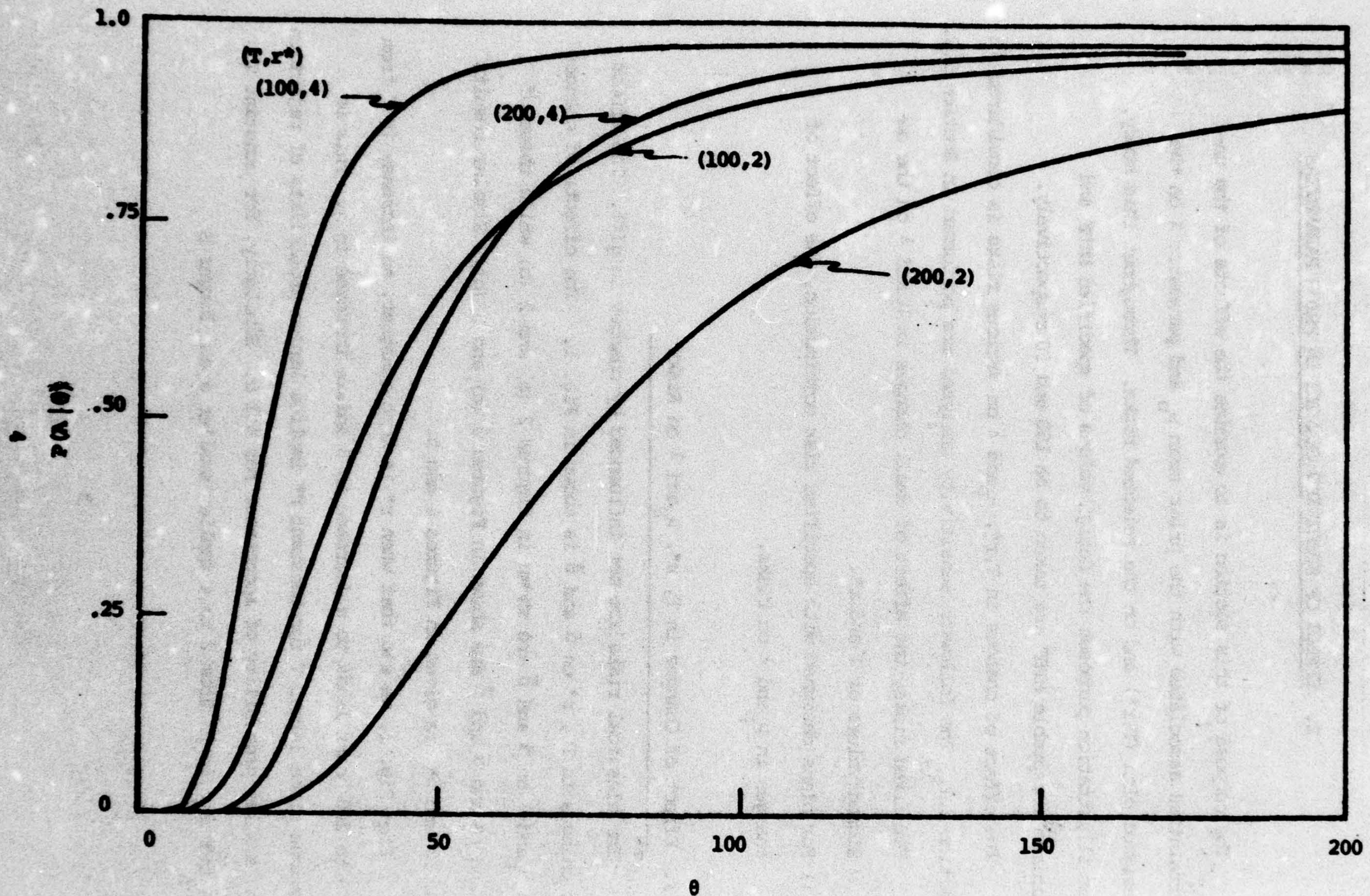

Fig. 1 Effect of Changes in (T,r*) on Classical O.C. Curves

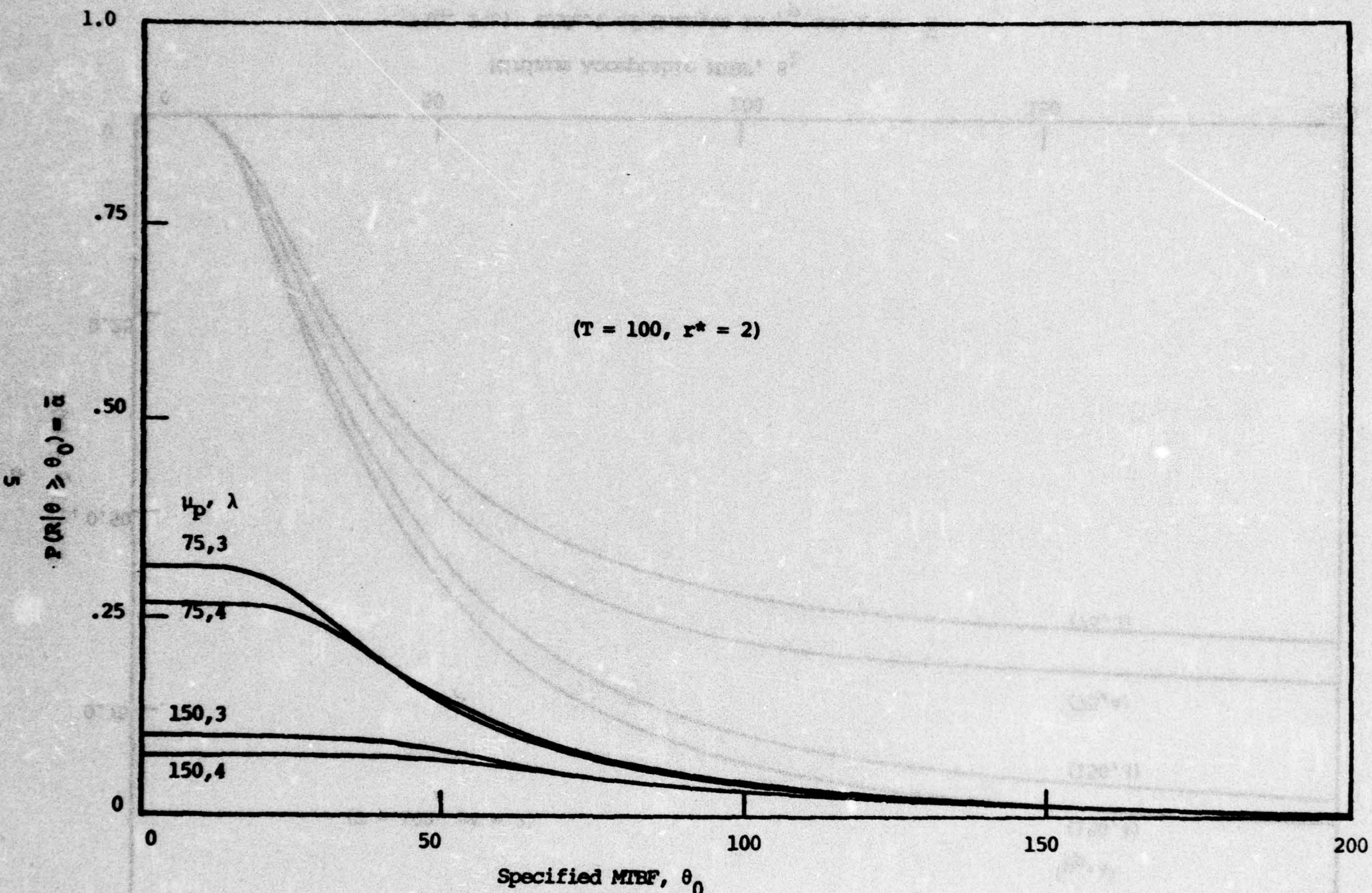

Fig. 2(a) Effect of Changes in μ_p and λ on $\bar{\alpha}$

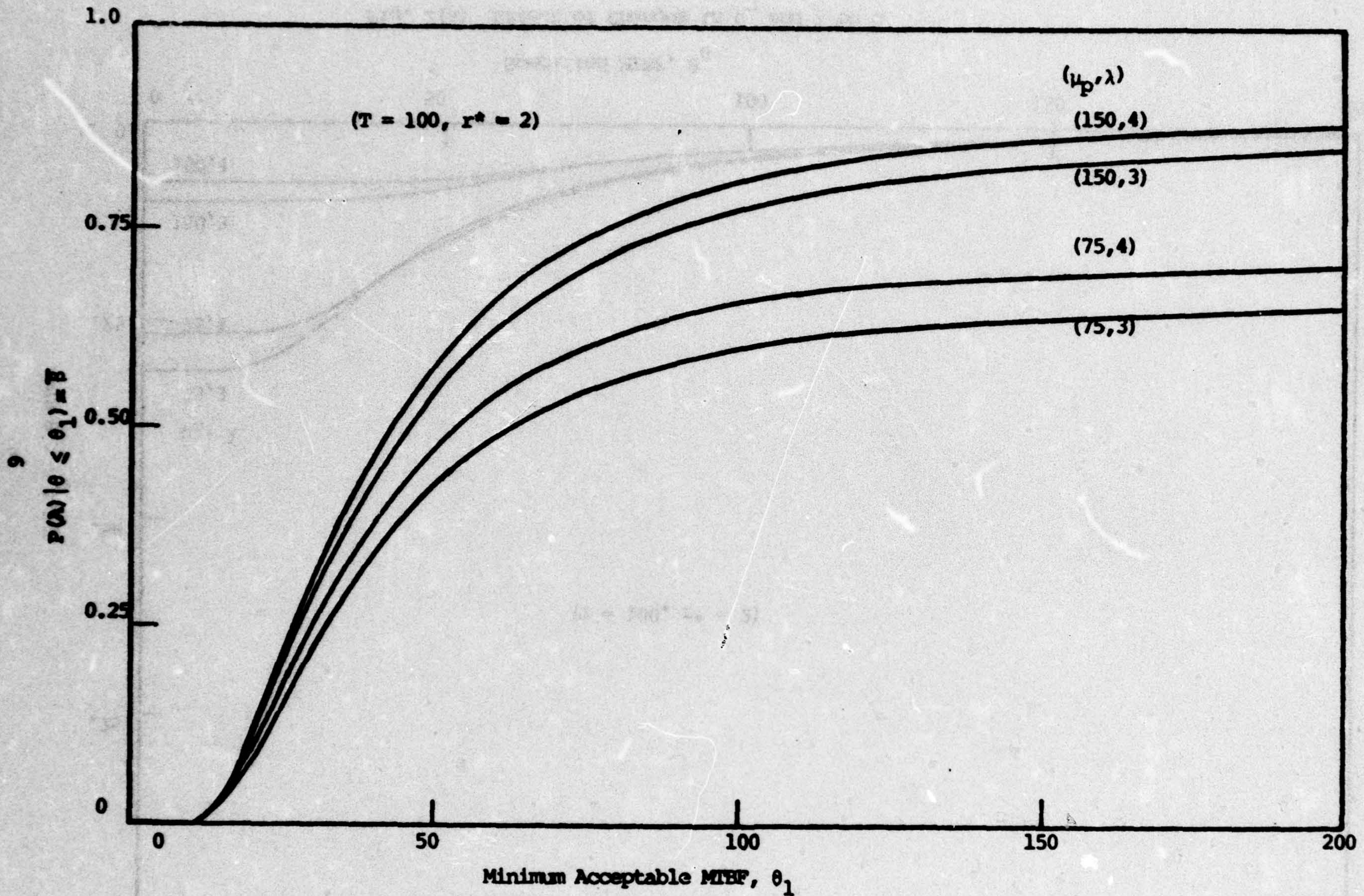

Fig. 2(b) Effect of Changes in μ_p and λ on $\bar{\beta}$

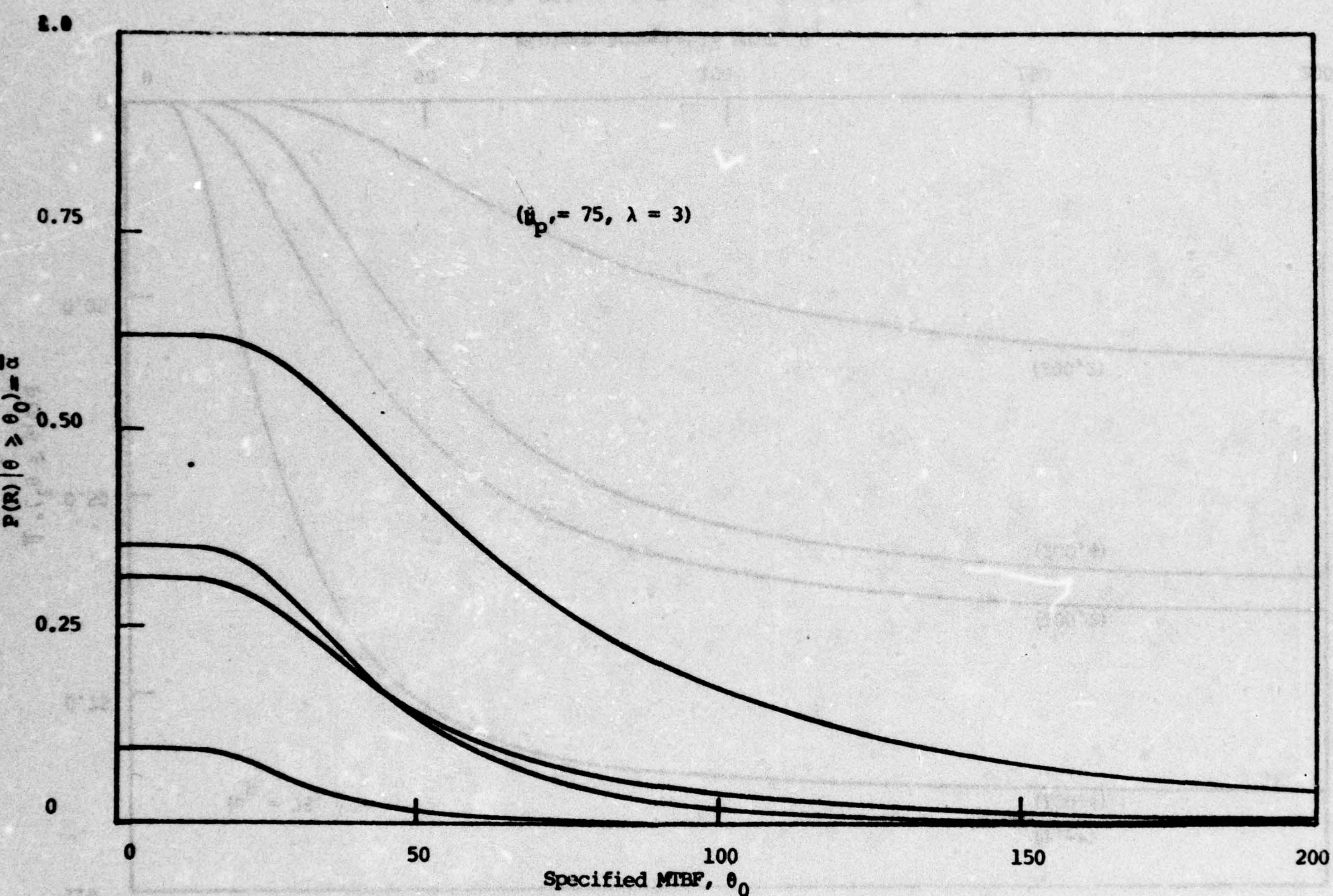

Fig. 3(a) Effect of Changes in T and r* on $\bar{\alpha}$

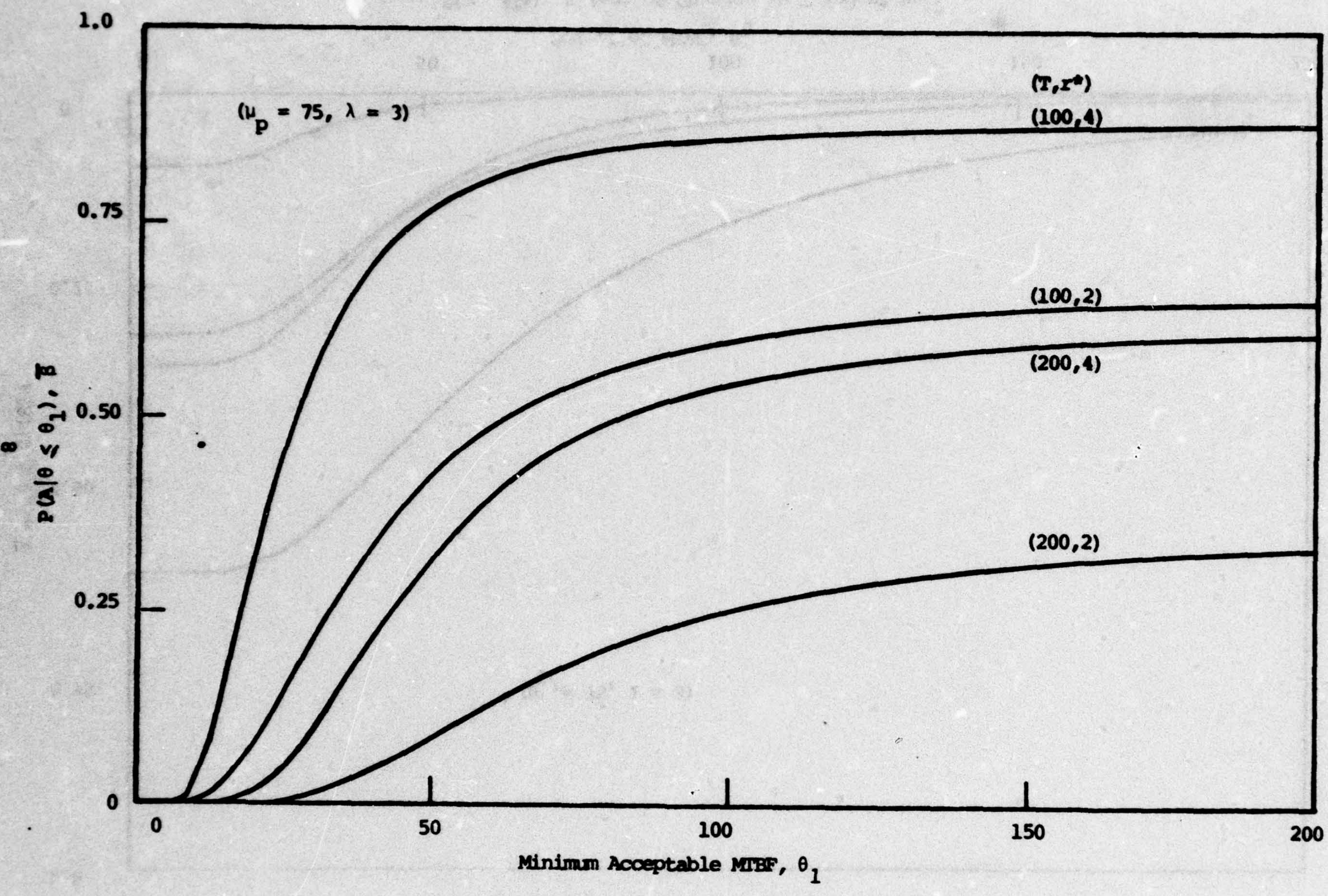

Fig. 3(b) Effect of Changes in T and r^* on $\bar{\beta}$

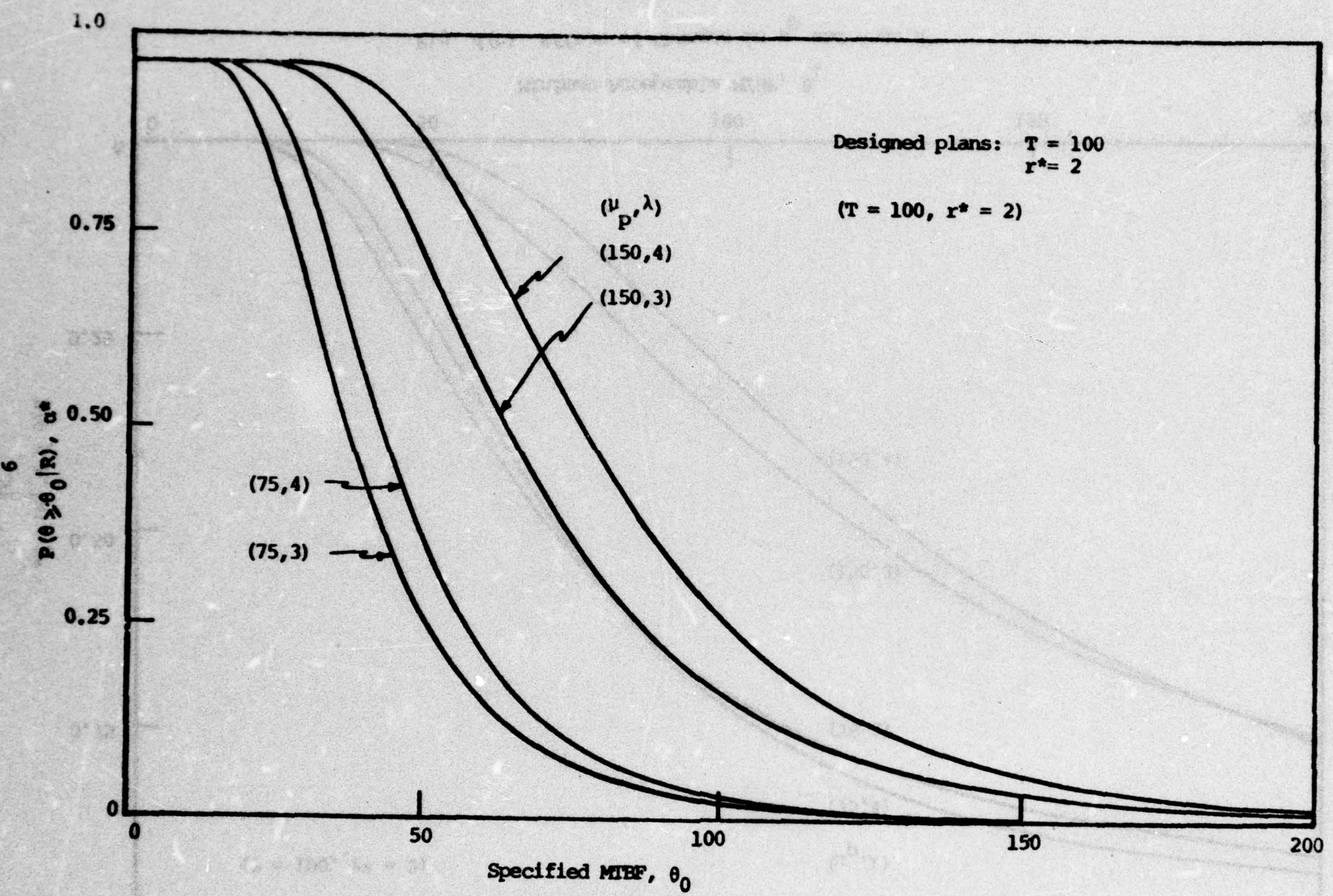

Fig. 4(a) Effect of Changes in μ_p and λ on α^*

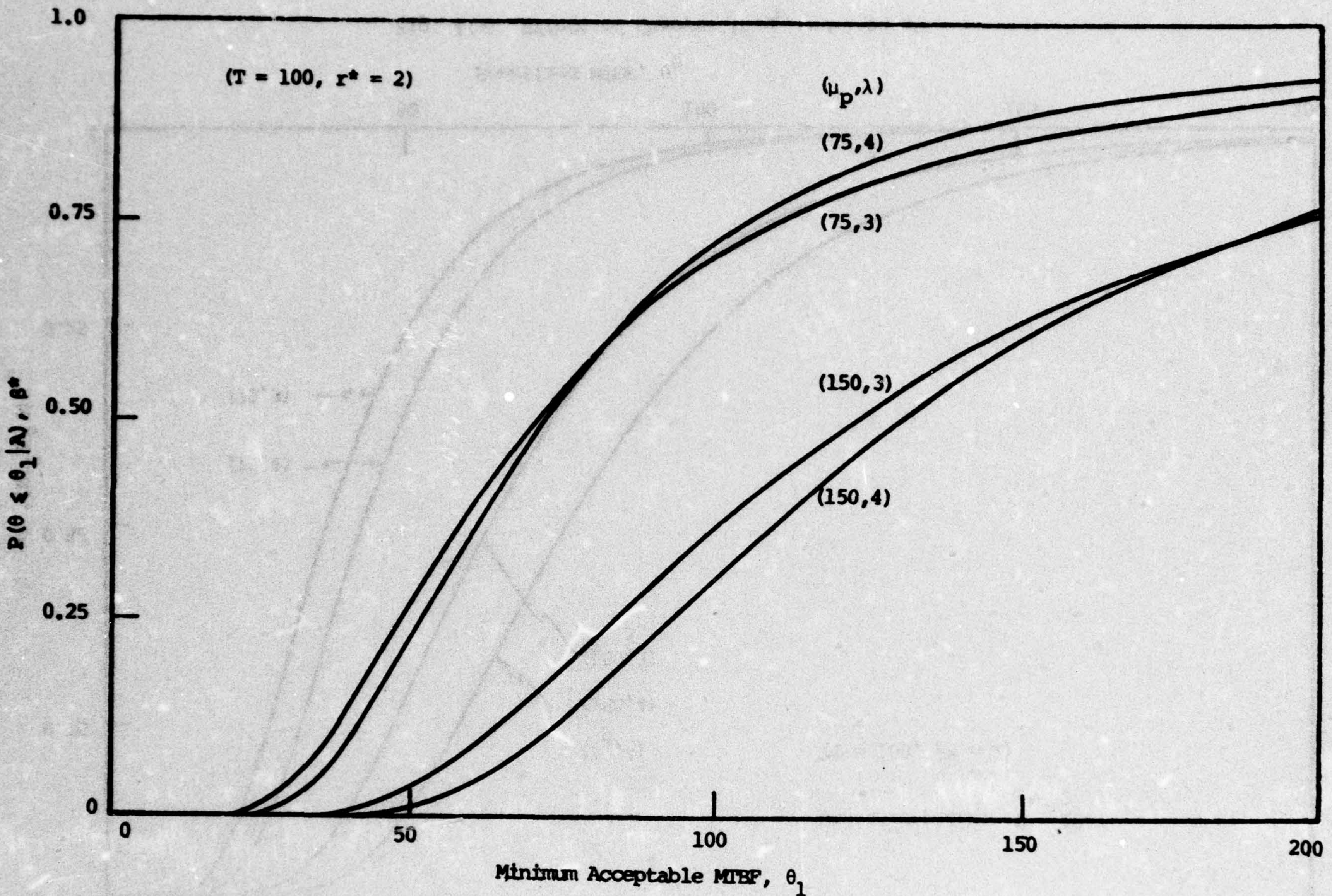

Fig. 4(b) Effect of Changes in μ_p and λ on β*

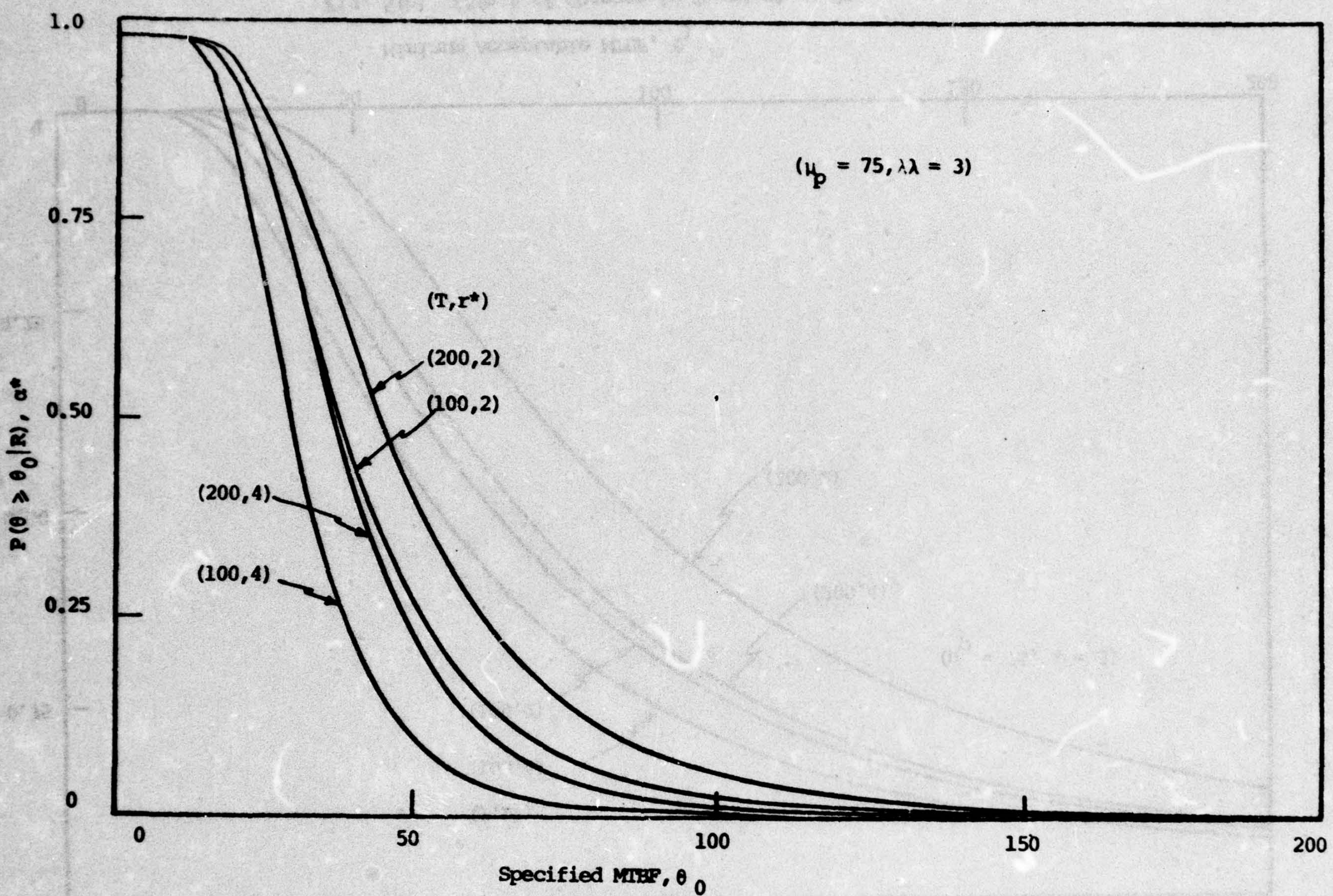

Fig. 5(a) Effect of Changes in T and r* on α^*

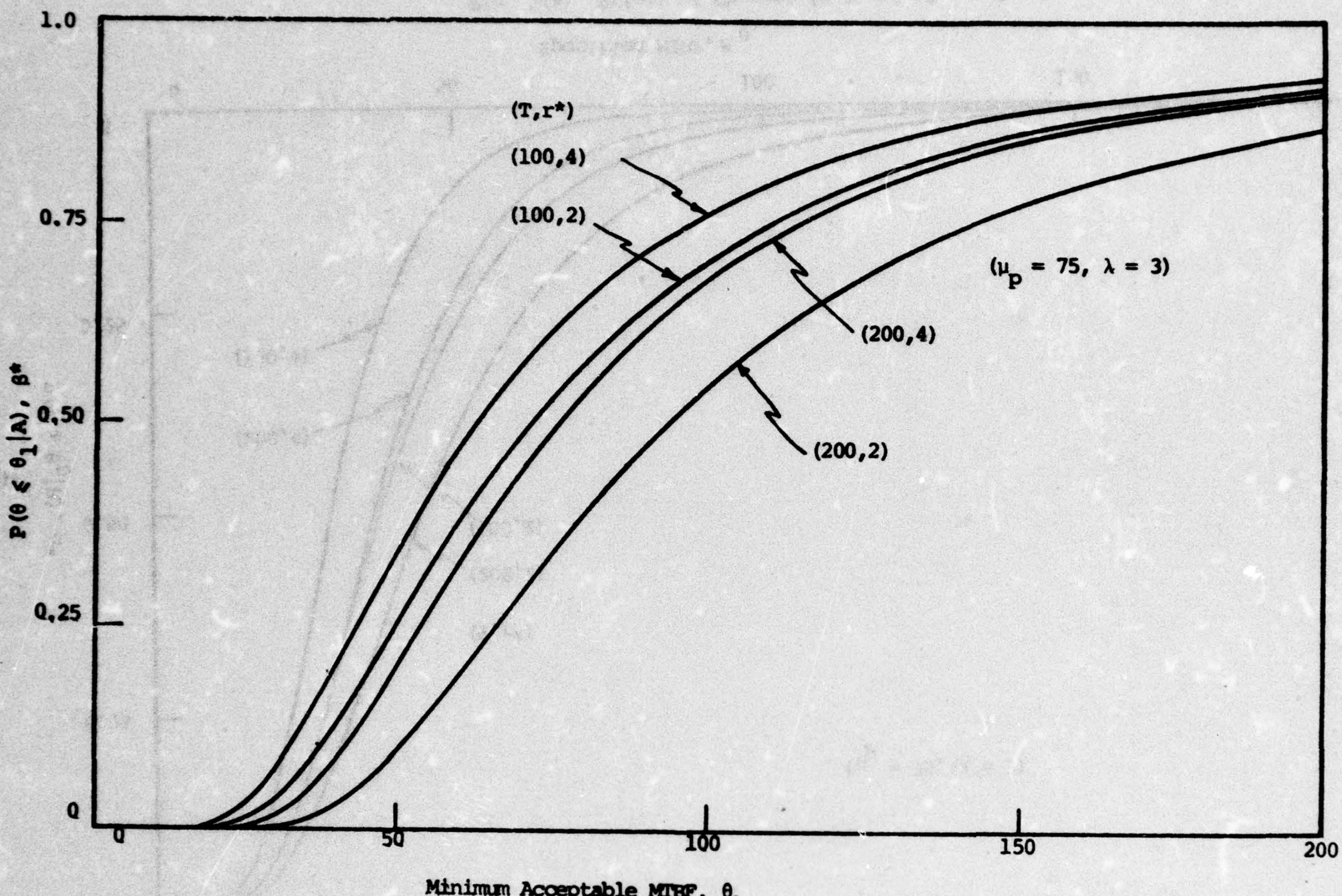

Fig. 5(b) Effect of Changes in T and r* on β*

From Fig. 2 (a), an increase in μ_p reduces $\bar{\alpha}$. The effect of change in λ on $\bar{\alpha}$ depends upon the value of θ_0. For small θ_0, an increase in λ reduces $\bar{\alpha}$ while the opposite occurs for larger values of θ_0. An increase in μ_p and λ implies larger $\bar{\beta}$ as seen from Fig. 2 (b). These results are true only within the experimental zone and may be explained as follows. $\bar{\alpha}$ and $\bar{\beta}$ are defined as

$$\bar{\alpha} = P(R|\theta \geq \theta_0) = \frac{P(\theta \geq \theta_0|R) \cdot P(R)}{P(\theta \geq \theta_0)}$$

$$\bar{\beta} = P(A|\theta \leq \theta_1) = \frac{P(\theta \leq \theta_1|A) \cdot P(A)}{P(\theta \leq \theta_1)}$$

An increase in μ_p and/or λ implies favorable prior, i.e., larger $P(A)$, and $P(\theta \geq \theta_0)$ and smaller $P(R)$ and $P(\theta \leq \theta_1)$. However, the changes in $P(\theta \geq \theta_0|R)$ and $P(\theta \leq \theta_1|A)$ depend upon the values of θ_0 and θ_1. Therefore, the effect of an increase in μ_p and λ on $\bar{\alpha}$ and $\bar{\beta}$ depends upon the values of θ_1 and θ_0. Usually the effect will be to increase $\bar{\beta}$ and decrease $\bar{\alpha}$.

From Figures 3 (a) and 3 (b), an increase in T for fixed r* and prior parameters implies larger $\bar{\alpha}$ and smaller $\bar{\beta}$. An increase in r* for constant T leads to a decrease in $\bar{\alpha}$ and an increase in $\bar{\beta}$. The reasoning behind these changes is the same as the one given above for the classical risks.

Similar inferences can be made regarding the influence of the prior parameters, μ_p and λ and of the designed plans (T,r*) on α^* and β^* from Figures 4 and 5 respectively. These inferences are valid within the range of parameter values considered.

2.2 Sensitivity of the Designed Plans and Computed Risks to Changes in Prior Parameters.

We now numerically evaluate the effect of changes in μ_p and λ on the designed T and r*, and the risks.

2.2.1. The Design Regions

The design regions being considered for this study are as follows. The primary design region considered in this study has four base points consisting of all combinations of $\mu_p = 75$, 125 and $\lambda = 3,5$. Around each base point, a factorial design is constructed with a 5% change in μ_p and 1 unit change in λ on either side of the base point. This implies that in the subsequent analysis the effect of a 10% change in μ_p and 2 unit change in λ will be evaluated. Additional factorial regions considered are: (1) $\mu_p = 120$, 140; $\lambda = 2.5, 3.5$. (2) $\mu_p = 120, 135$; $\lambda = 2.5, 3.5$. (3) $\mu_p = 200$, 300; $\lambda = 2.5, 3.5$ and (4) $\mu_p = 187.5, 262.5$; $\lambda = 2.5, 3.5$.

It should be noted that these regions have been chosen for illustration purposes. Similar analyses could be done for other regions of interest.

The choice of the changes in μ_p and λ is governed by the uncertainties associated with the prior parameters. Figures 6(a) to 6(h) show the plots of the inverted gamma prior density for each of the five points of the eight factorial regions in this study. The plots indicate that the selection of changes in μ_p and λ is reasonable (except for designs 7 and 8 in Figures 6(g) and 6(h)). In other words, if the prior is based upon a reasonable amount of data, the changes in μ_p and λ considered in this study would be of the order of $2\sigma_{\hat{\mu}_p}$ and $2\sigma_{\hat{\lambda}}$ respectively. It is suggested that this be further verified by conducting an analysis of the real data.

Table 1 gives the values of the probabilities $P(\theta \leq \theta_1)$, $P(\theta_1 < \theta < \theta_0)$ and $P(\theta \geq \theta_0)$ for the forty prior distributions considered above. From this table we note that, in general, the probabilities in the tail regions $\theta \geq \theta_0$ and $\theta \leq \theta_1$ change considerably with changes in μ_p and λ. This indicates that the designed plans , especially those that use these areas as risk criteria, may be sensitivie to changes in the prior parameters.

2.2.2 Changes in Designed Plans For Fixed Risks

For purposes of this analysis, plans were designed for specified risks and parameters μ_p and λ. The results of this analysis are shown in Tables 2 through 7. In these tables, the center point shows the designed plans (T,r^*) for the specified risk values and the corresponding prior parameters. The values at the corners of the various squares are the designed plans if

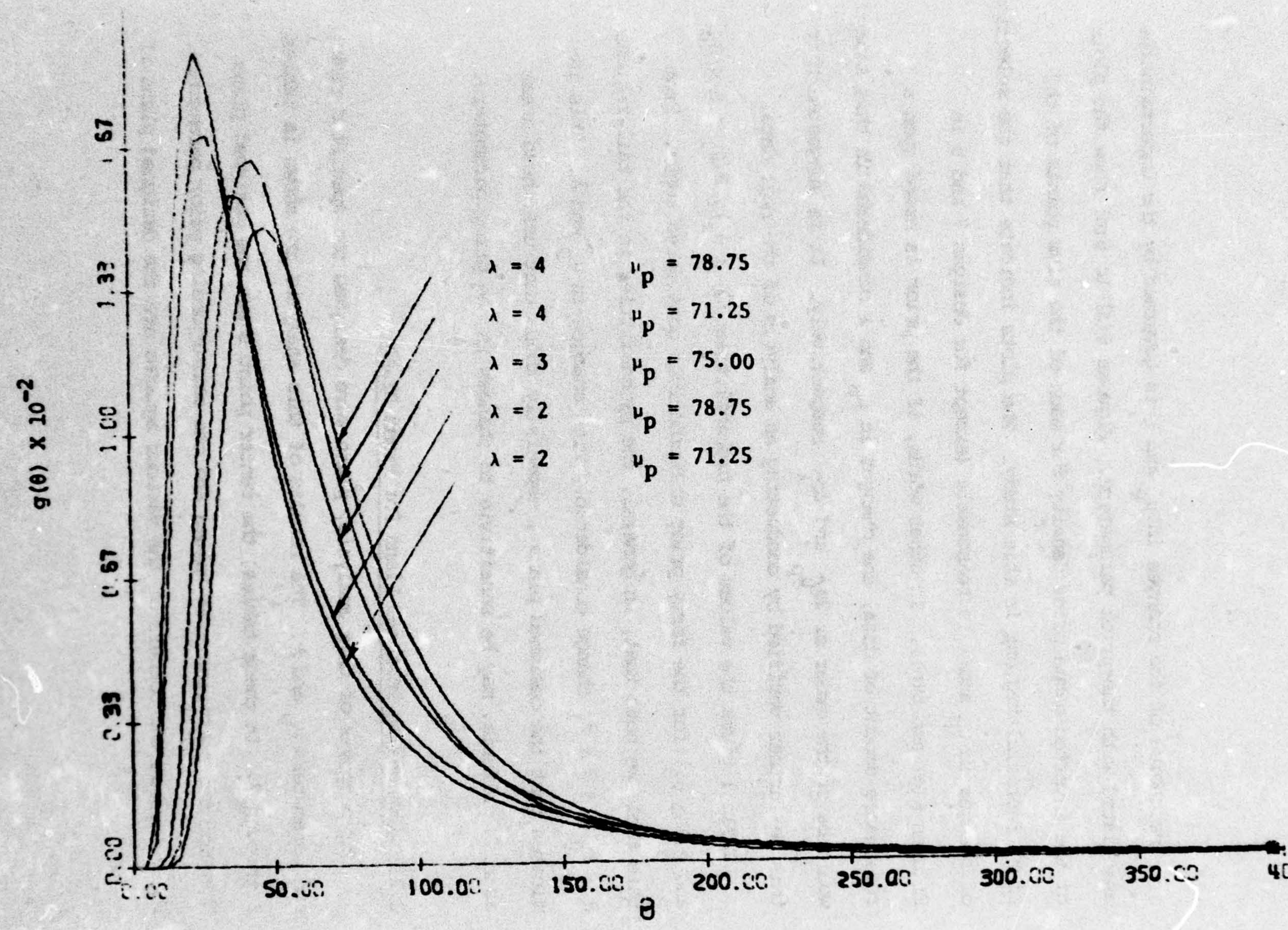

Fig. 6(a) Inverted Gamma Density

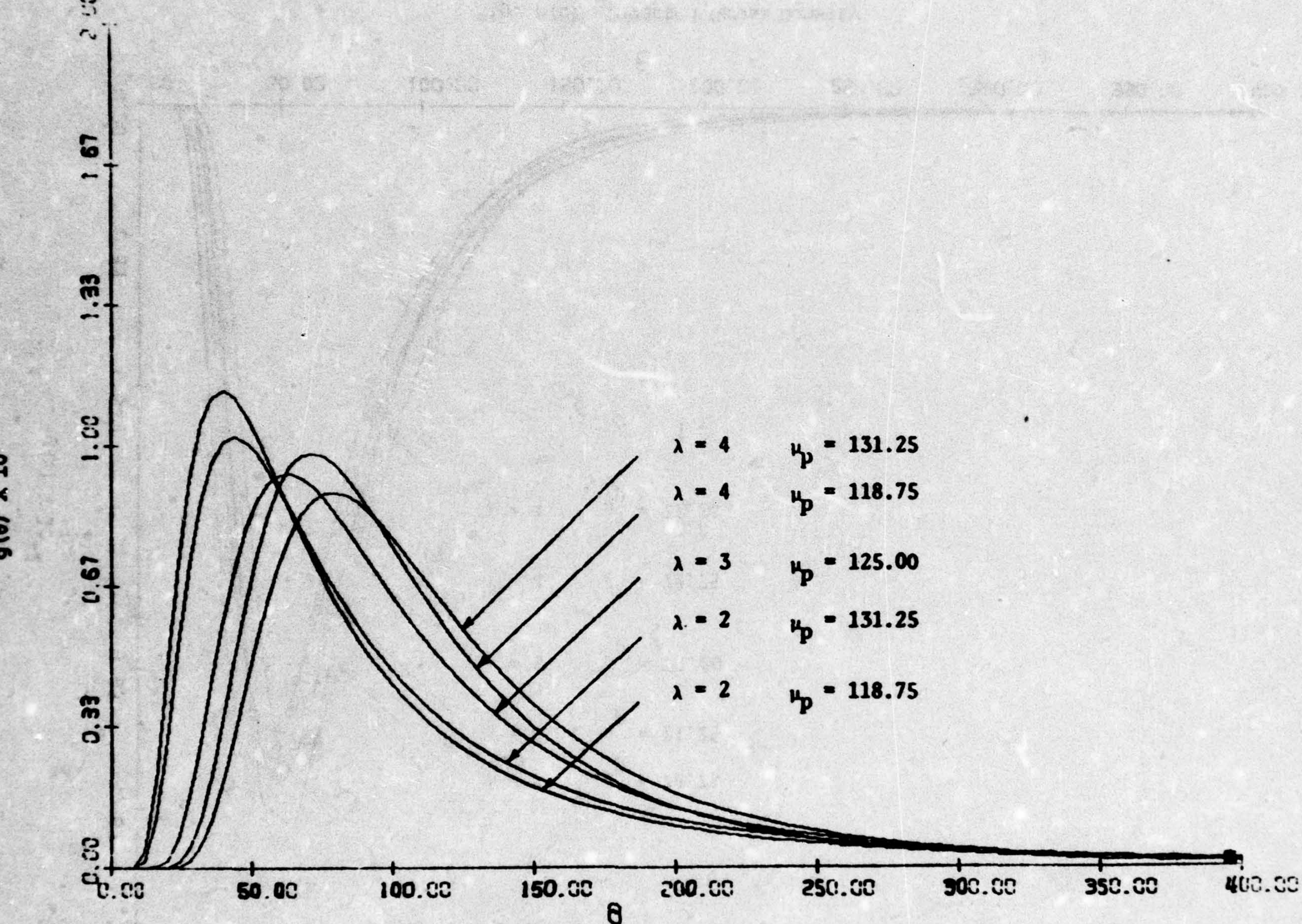

Fig. 6(b) Inverted Gamma Density

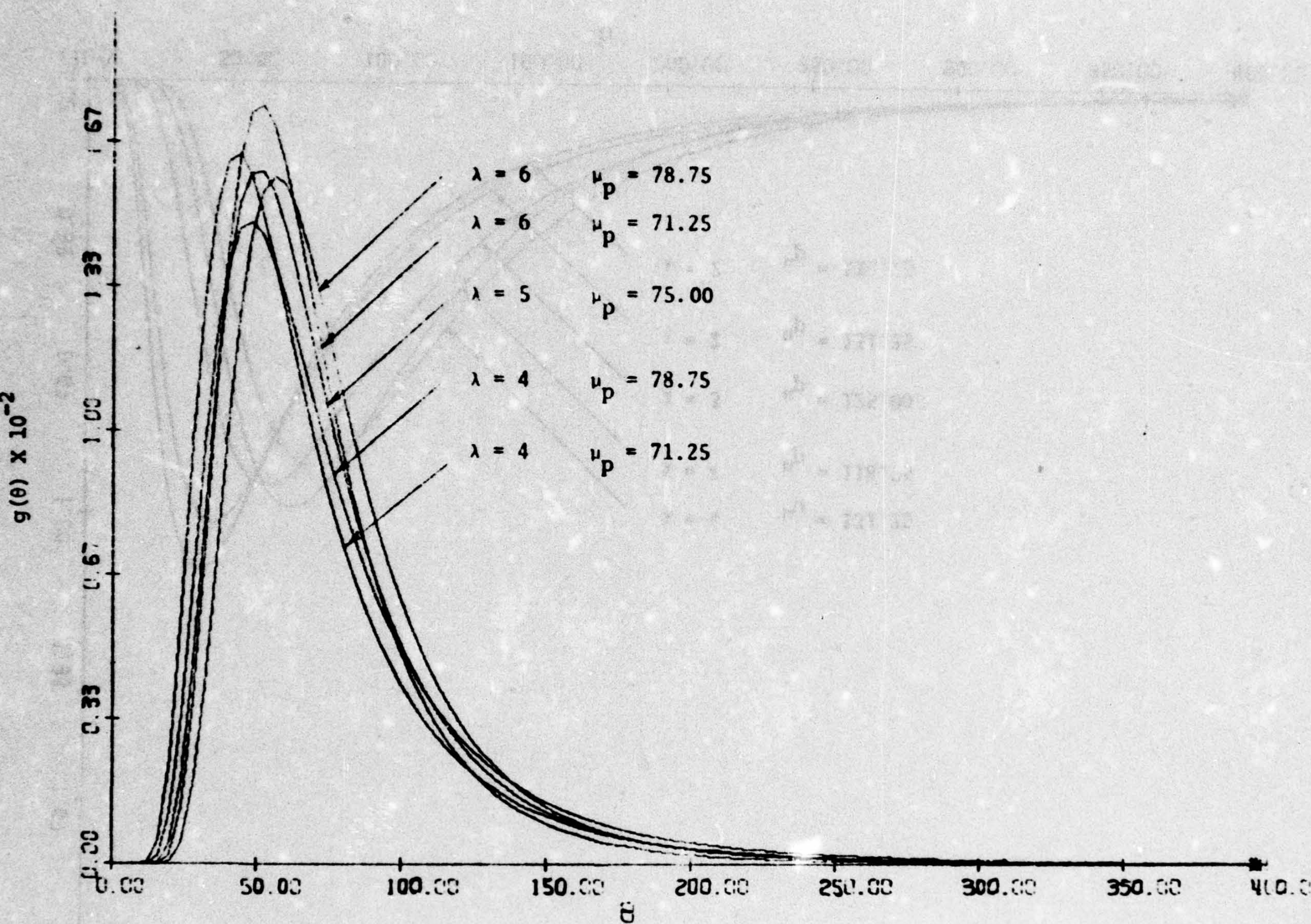

Fig. 6(c) Inverted Gamma Density

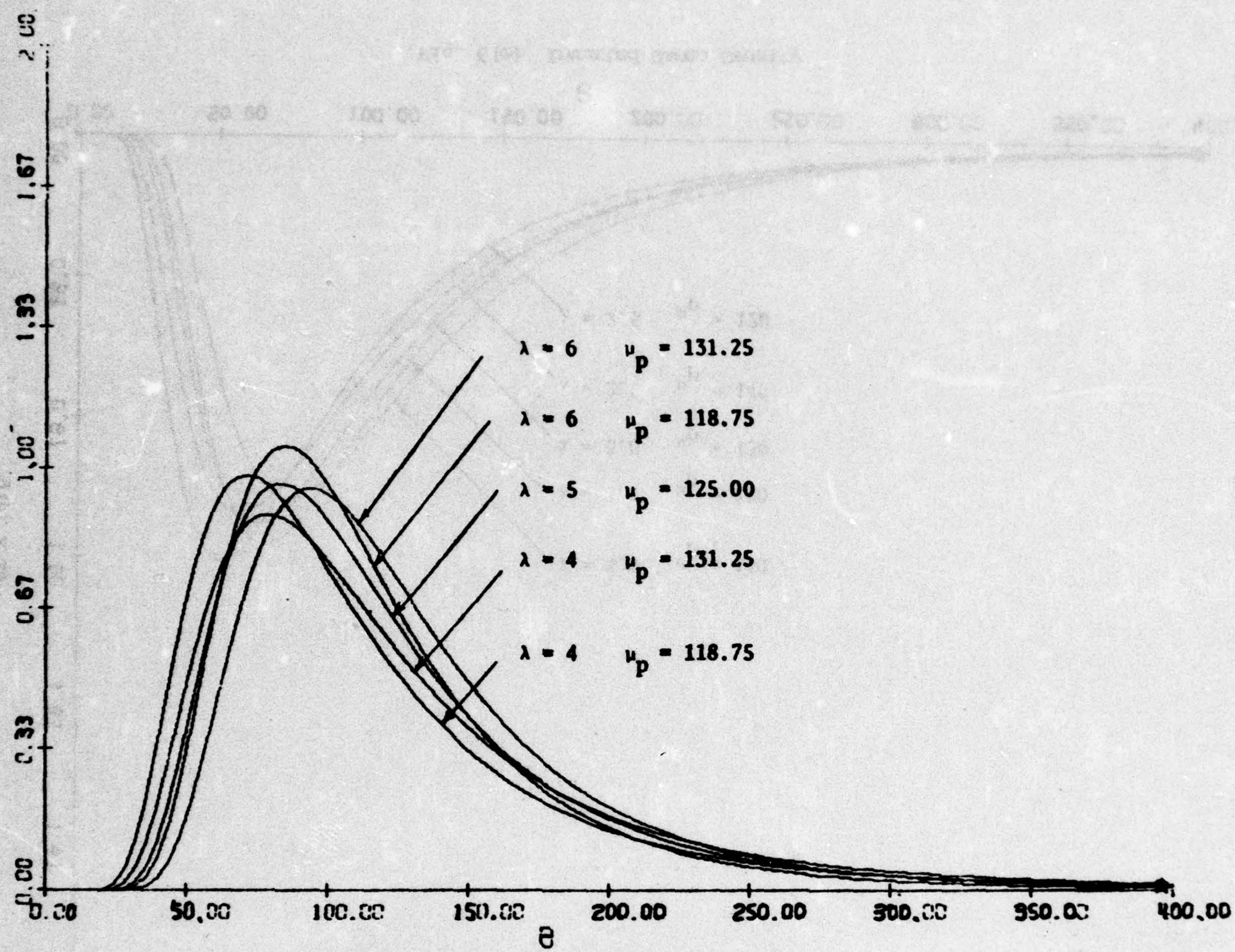

Fig. 6(d) Inverted Gamma Density

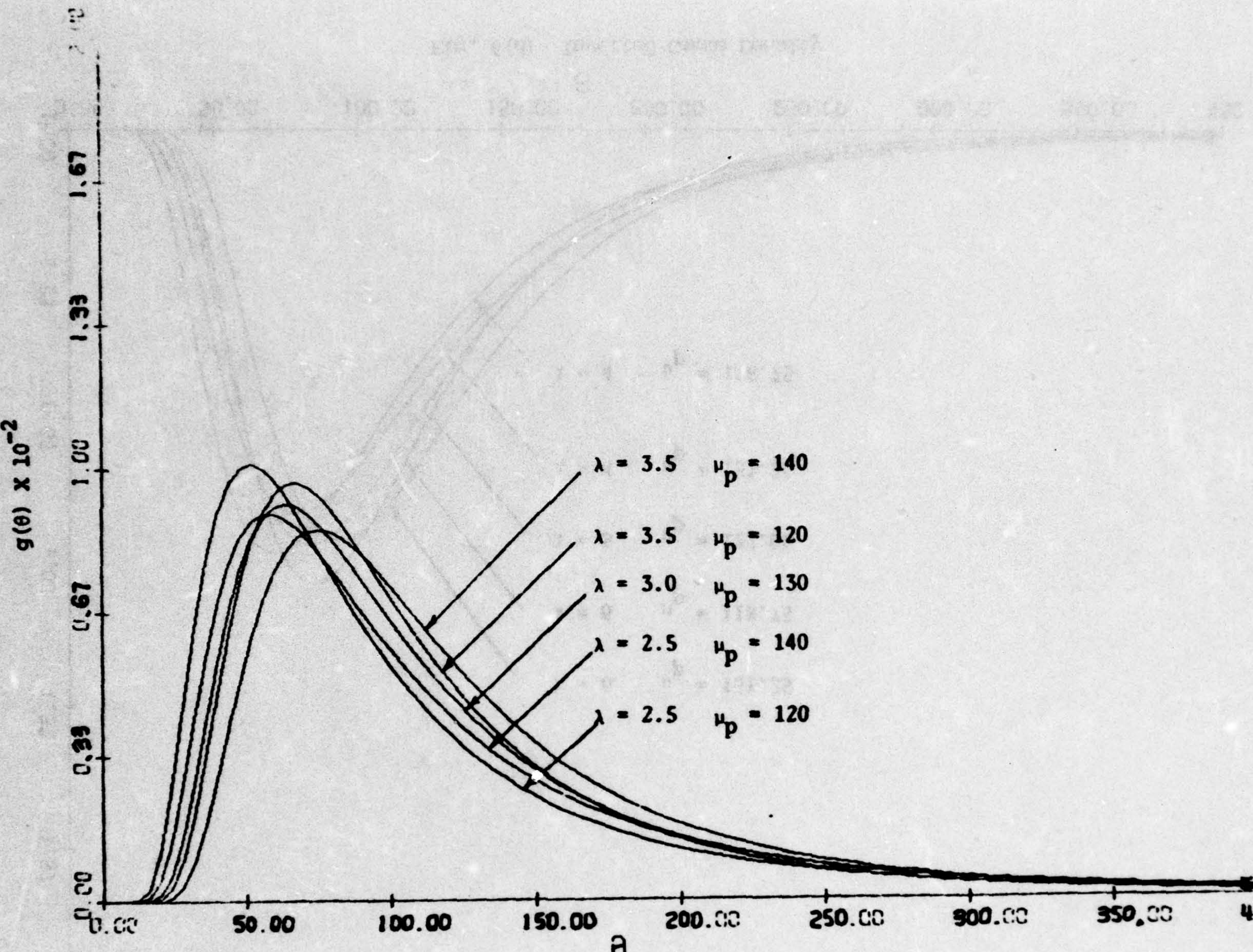

Fig. 6(e) Inverted Gamma Density

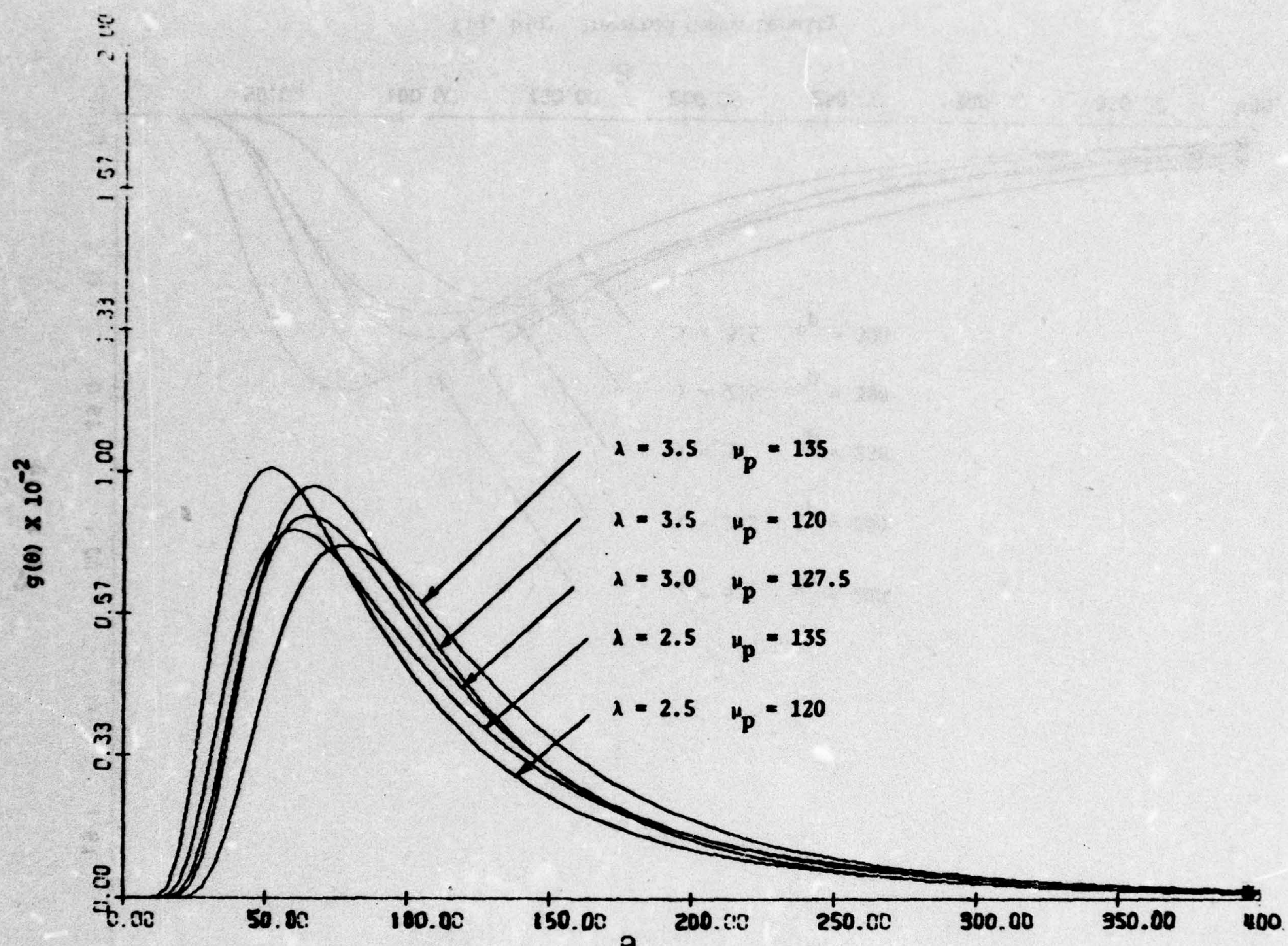

Fig. 6(f) Inverted Gamma Density

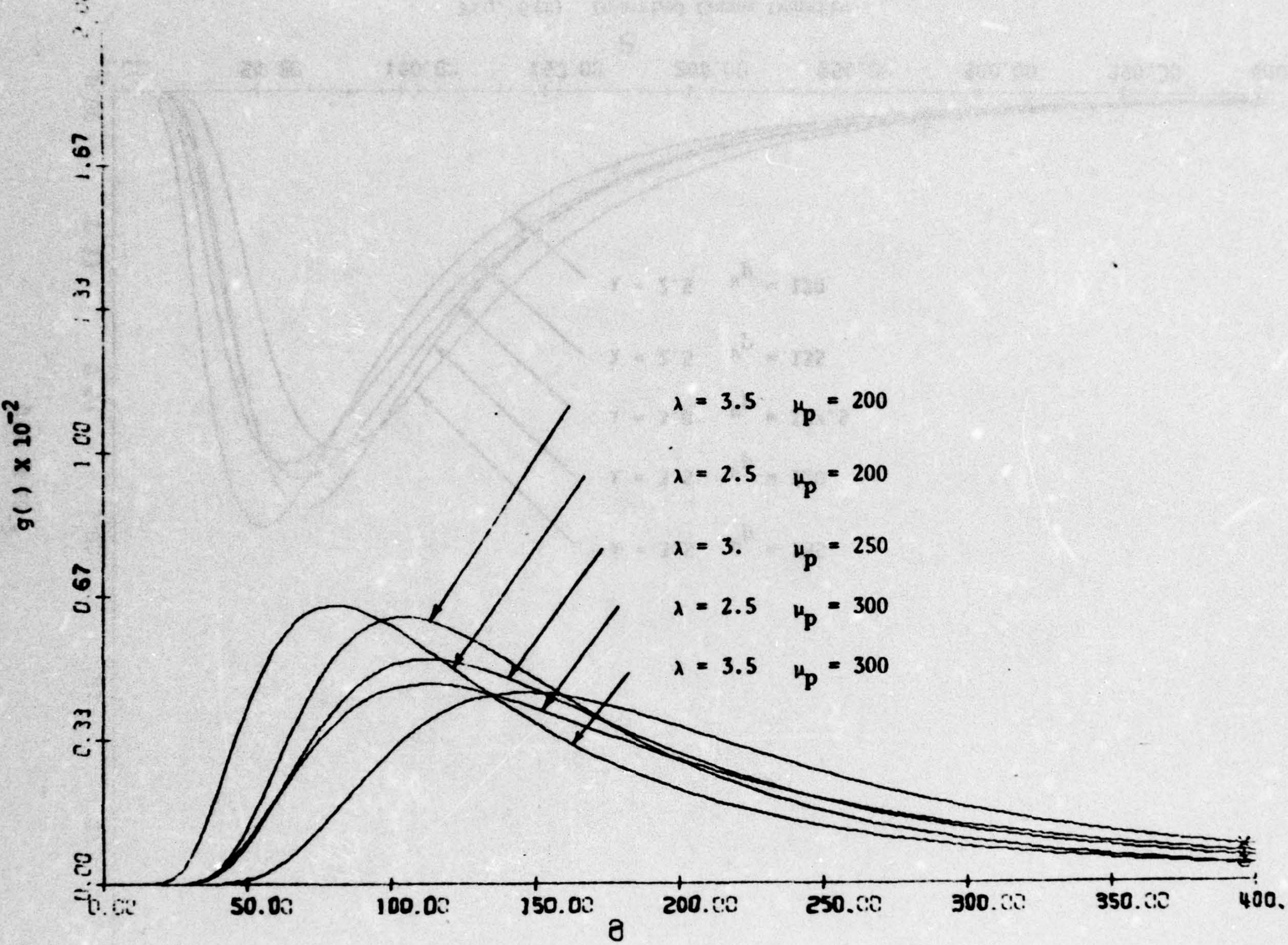

Fig. 6(g) Inverted Gamma Density

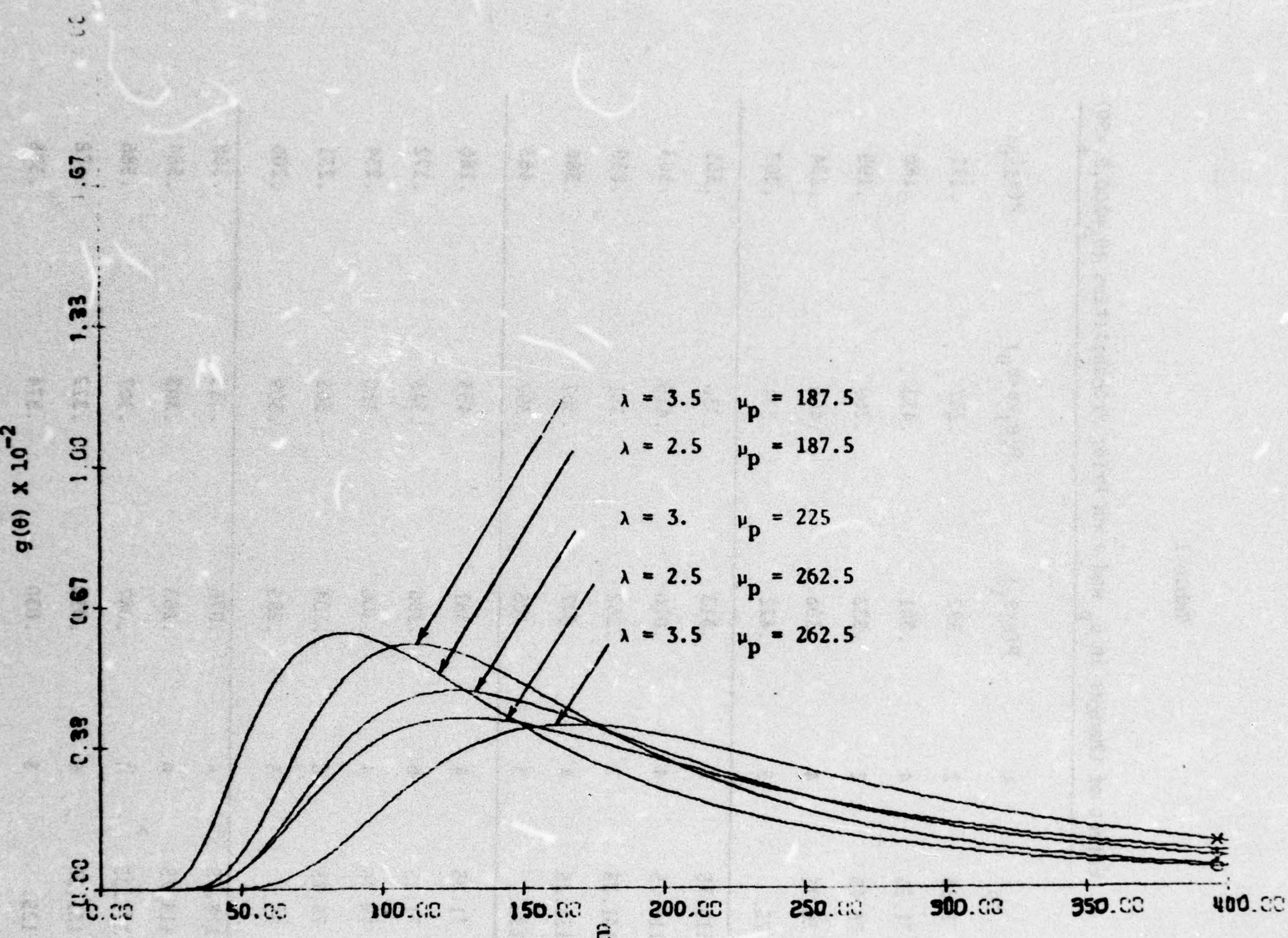

Fig. 6(h) Inverted Gamma Density

Table 1

Effect of Changes in μ_p and λ on Prior Probabilities (θ_o=100,θ_1=50)

μ_p	λ	$P(\theta \leq \theta_1)$	$P(\theta_1 < \theta < \theta_0)$	$P(\theta \geq \theta_0)$
71.25	2	.582	.247	.171
71.25	4	.381	.433	.186
78.75	2	.532	.269	.199
78.75	4	.306	.460	.234
75.	3	.422	.371	.207
118.75	2	.313	.336	.351
118.75	4	.076	.415	.508
131.25	2	.262	.351	.839
131.25	4	.047	.367	.586
125.	3	.125	.392	.483
71.25	4	.381	.433	.186
71.25	6	.286	.543	.172
78.75	4	.306	.460	.234
78.75	6	.204	.565	.231
75.	5	.285	.509	.206
118.75	4	.076	.415	.508
118.75	6	.203	.393	.584
131.25	4	.047	.367	.586
131.25	6	.010	.312	.678
125.	5	.030	.374	.595

Table 1 (Continued)

μ_p	λ	$P(\theta<\theta_1)$	$P(\theta_1\le\theta<\theta_0)$	$P(\theta\ge\theta_0)$
120.	2.5	.206	.380	.414
120.	3.5	.101	.609	.489
140.	2.5	.136	.361	.503
140.	3.5	.052	.347	.601
130.	3.	.109	.382	.509
120.	2.5	.206	.380	.414
120.	3.5	.101	.409	.489
135.	2.5	.151	.367	.482
135.	3.5	.062	.364	.574
127.	3.	.117	.387	.496
200.	2.5	.035	.247	.717
200.	3.5	.005	.160	.832
300.	2.5	.003	.092	.905
300.	3.5	.000	.029	.971
250.	3.	.003	.105	.892
187.5	2.5	.047	.273	.680
187.5	3.5	.009	.193	.798
262.5	2.5	.007	.138	.855
262.5	3.5	.000	.057	.943
225.	3.	.007	.147	.846

Table 2

Effect of Changes in μ_p and λ on T and r*: Criterion (α,β^*)

$(\theta_1=50,\ \theta_o=100)$

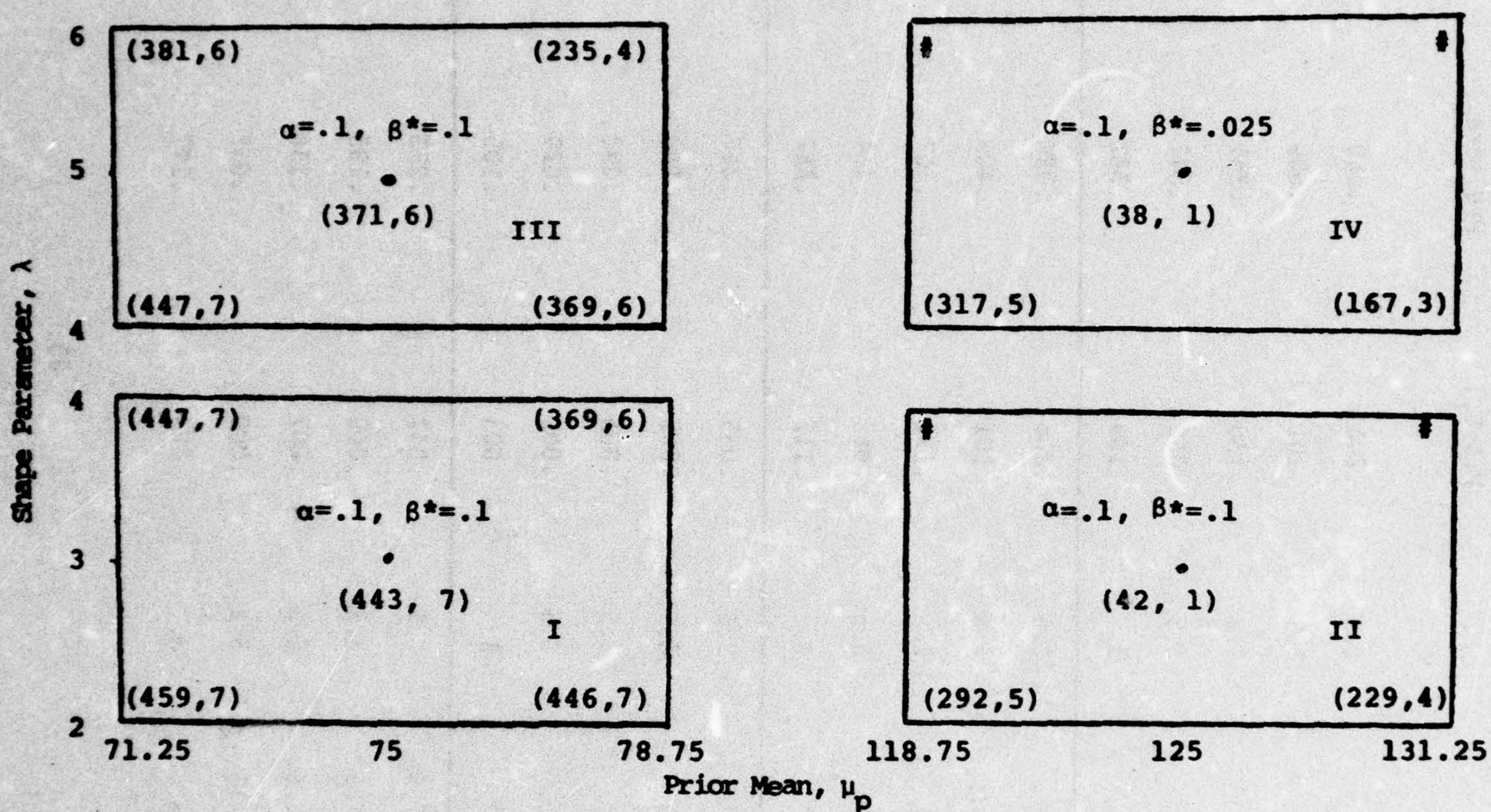

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Table 3

Effect of Changes in μ_p and λ on T and r*: Criterion $(\overline{\alpha},\overline{\beta})$

$(\theta_1=50,\ \theta_o=100)$

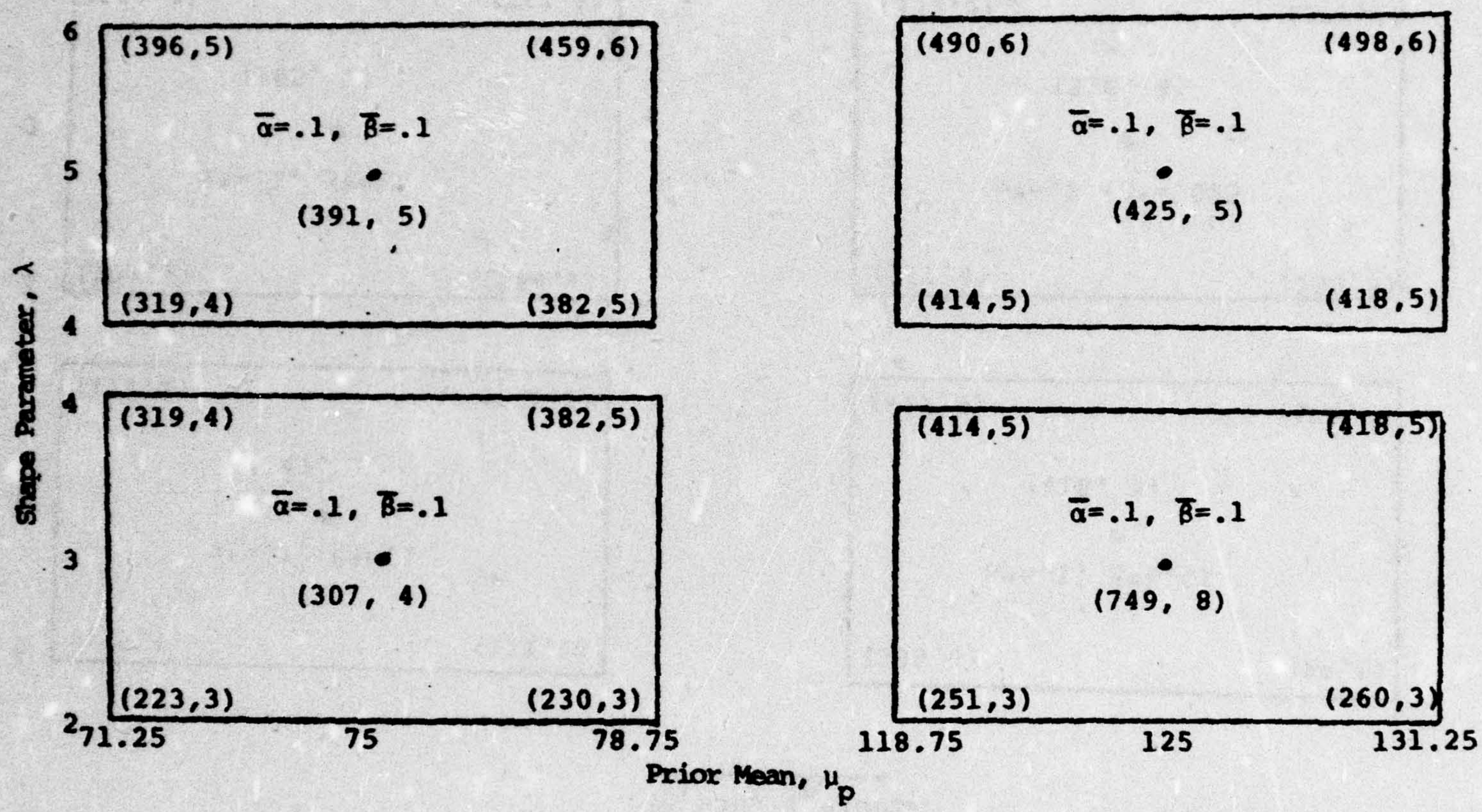

Table 4

Effect of Changes in μ_p and λ on T and r*: Criterion (α^*, β^*)

$(\theta_1 = 50, \ \theta_o = 100)$

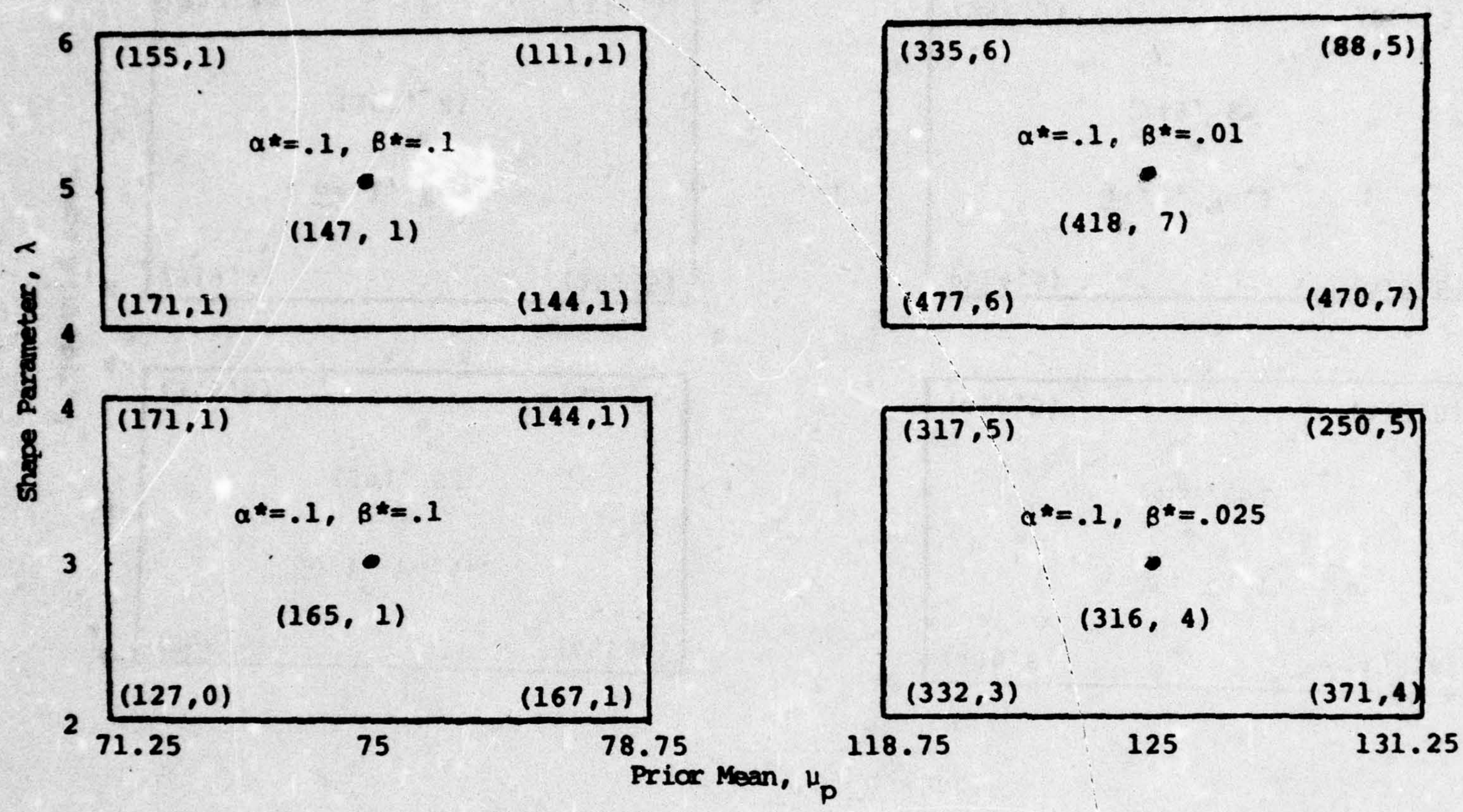

Table 5

Effect Changes in μ_p and λ on T and r*: Criterion $(\bar{\alpha},\beta^*)$

$(\theta_1 = 50,\ \theta_0 = 100)$

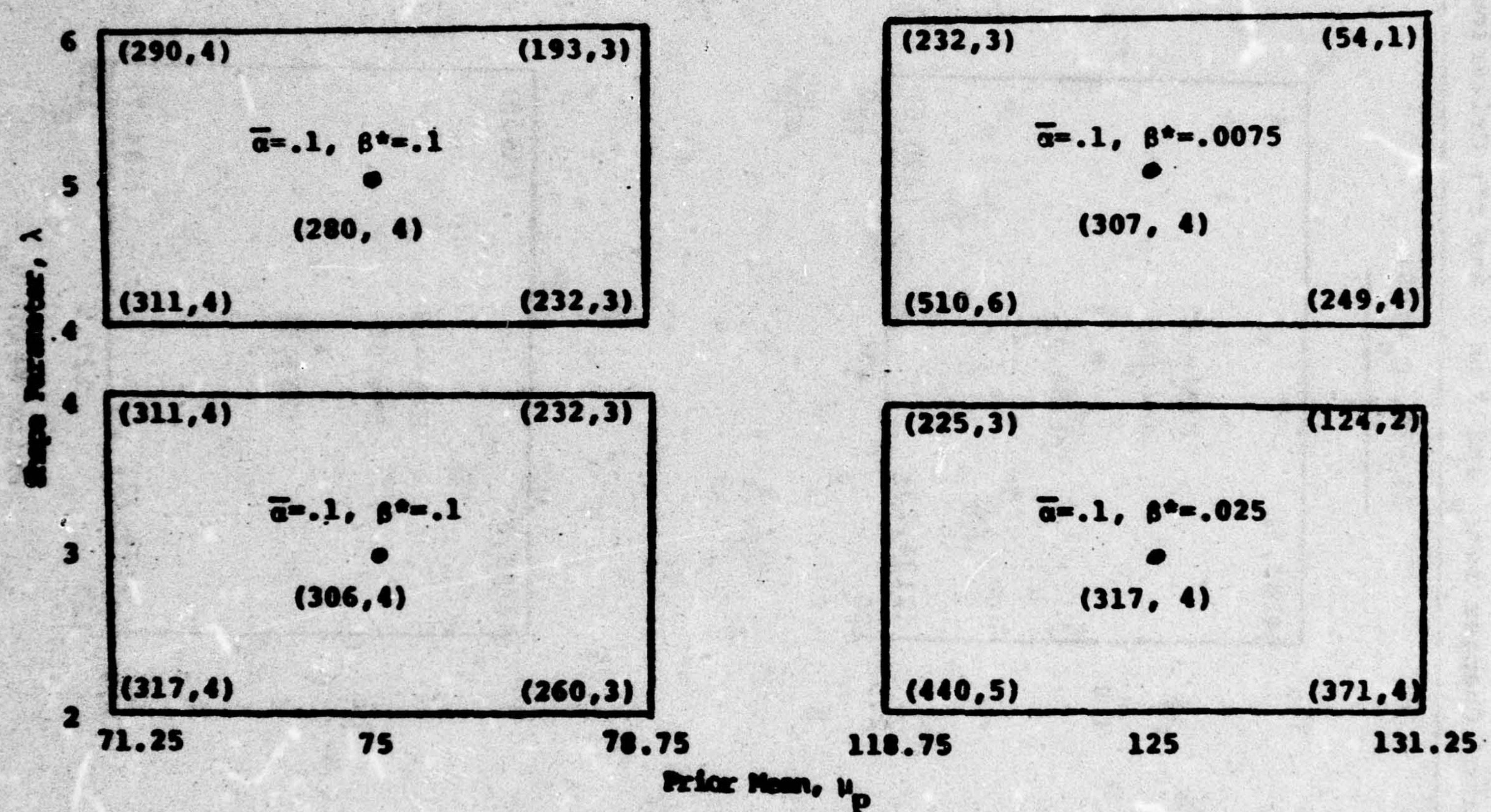

Table 6

Effect of Changes in μ_p and λ on T and r*: Criterion (P(A), β*)

$(\theta_1=50,\ \theta_o=100)$

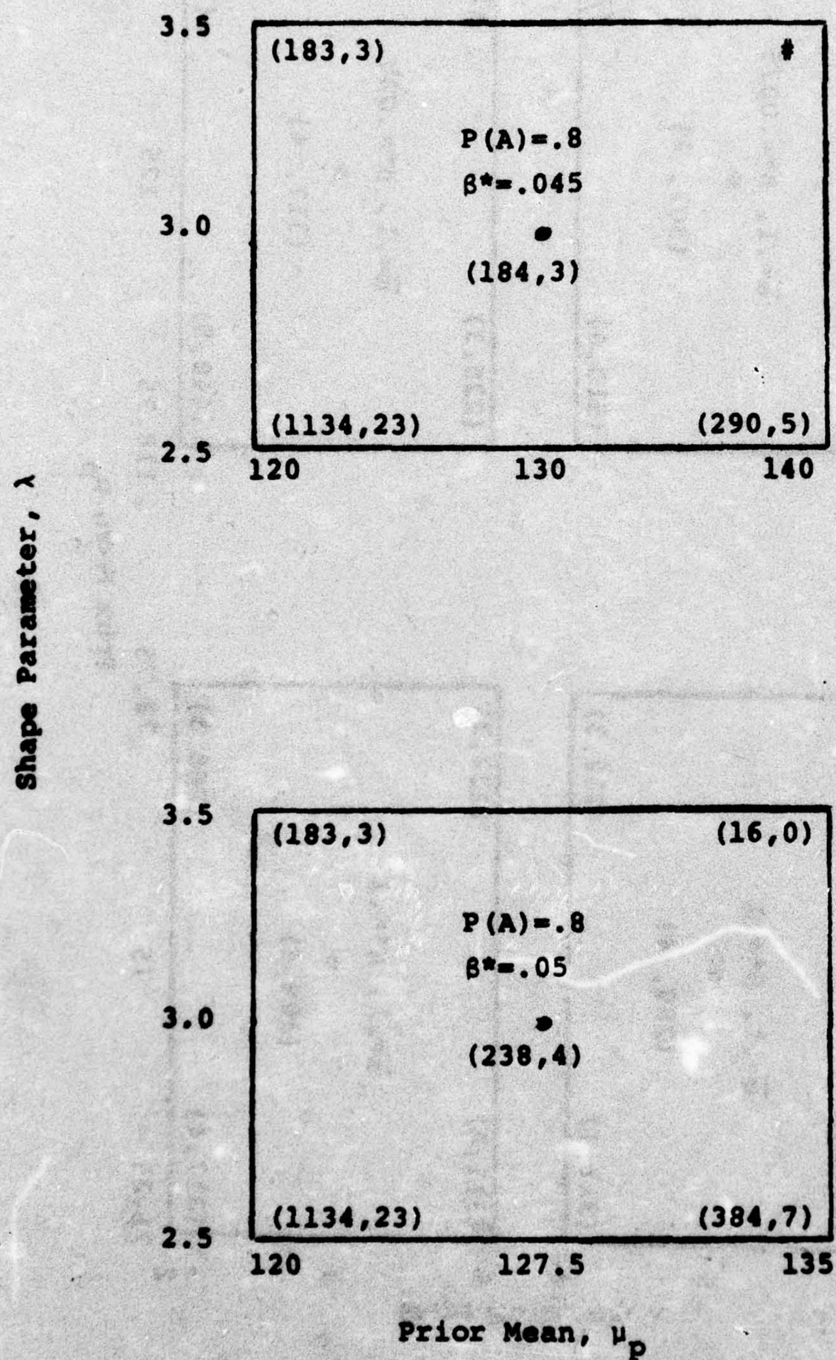

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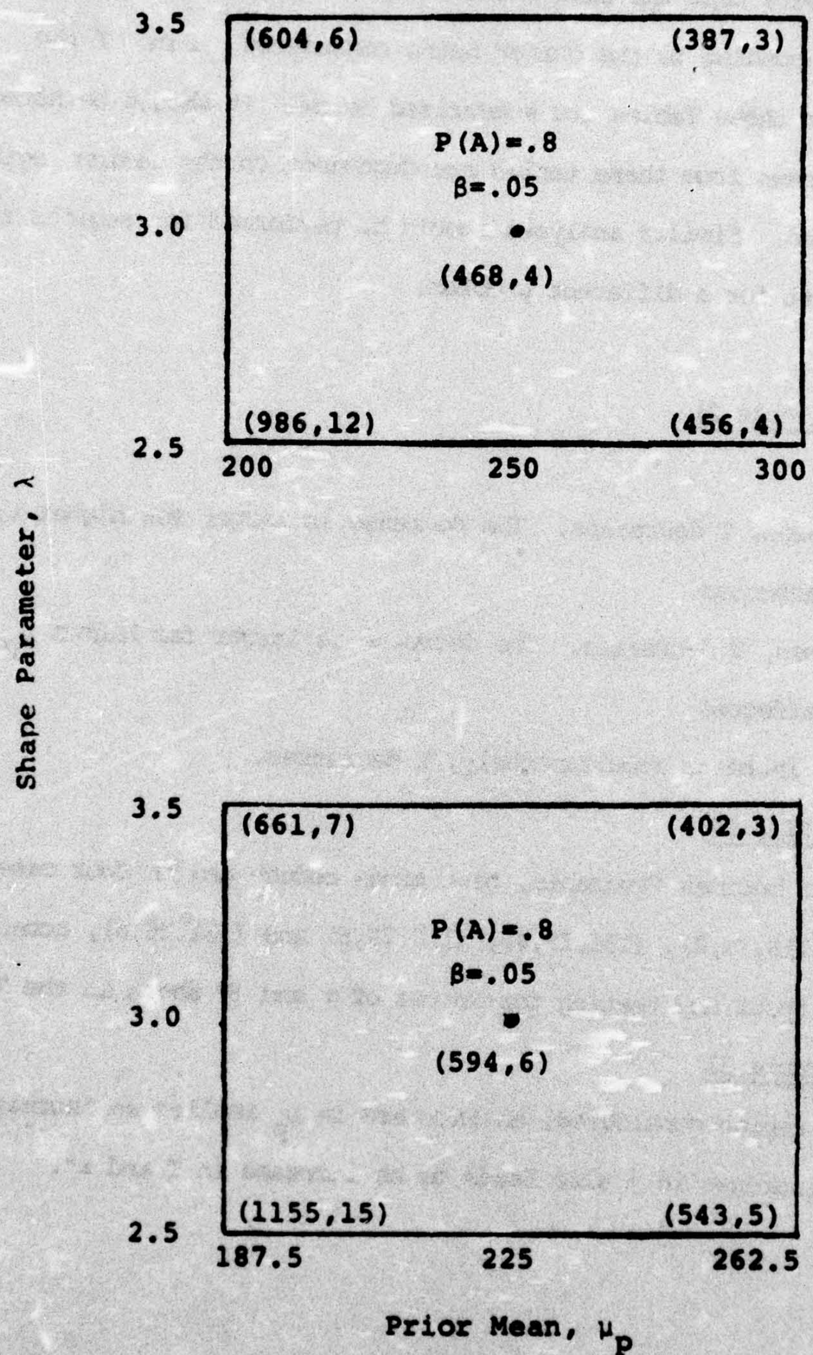
Table 7

Effect of Changes in μ_p and λ on T and r*: Criterion (P(A),β)

(θ_1=50, θ_o=100)



the risk values were kept the same while the prior parameters were changed to the values corresponding to the corner being considered. Some of the observations from these Tables are summarized below. It should be noted that the inferences drawn from these tables are dependent on the design region being investigated. Similar analyses should be performed for regions that may be of interest for a different problem.

<u>Fixed α and β^* (Table 2)</u>

<u>Factorial I</u>

As μ_p increases, T decreases. The decrease is larger for higher λ; r^* is almost unaffected.

As λ increases, T decreases. The decrease is larger for higher μ_p. r^* is almost unaffected.

As μ_p and λ increase simultaneously, T decreases.

<u>Factorials II, III, IV</u>

As the prior becomes favorable, test times reduce and in four cases, viz. (μ_p,λ) = (118.75,4), (131.25,4), (118.75,6) and (131.25,6), acceptance is permitted without any testing for values of α and β^* shown in the Table.

<u>Fixed $\bar{\alpha}$ and $\bar{\beta}$(Table 3)</u>

Within the region considered, an increase in μ_p implies an increase in T and r^*. An increase in λ also leads to an increase in T and r^*.

Fixed α^* and β^* (Table 4)

Depending upon the values of μ_p and λ, and increase in μ_p may lead to an increase or a decrease in T and r*. Similarly, an increase in λ may lead to larger or smaller T and r*.

Fixed $\bar{\alpha}$ and β^* (Table 5)

An increase in μ_p leads to a reduction in T and r* for constant λ. An increase in λ may yield larger or smaller T and r* depending upon the parameter values.

Fixed P(A) and β^* (Table 6)

An increase in μ_p and/or λ leads to a reduction in T and r*.

Fixed P(A) and β (Table 7)

An increase in μ_p and/or λ reduces T and r*.

2.2.3 Changes in Computed Risks For Fixed Plans

The design region described in Section 2.2.1 was considered for numerically studying the effects of changes in μ_p and λ on the risks. The risks were evaluated at the four corner points of the design assuming that the plan designed for the center point is also employed at the corresponding corner points. The results are shown in Tables 8 to 13 and the effects of μ_p and λ on the risks can be observed from these tables.

Table 8

Effect of Changes in μ_p and λ on Risks: Criterion $(\alpha, \beta+)$

$(\theta_1=50,\ \theta_o=100)$

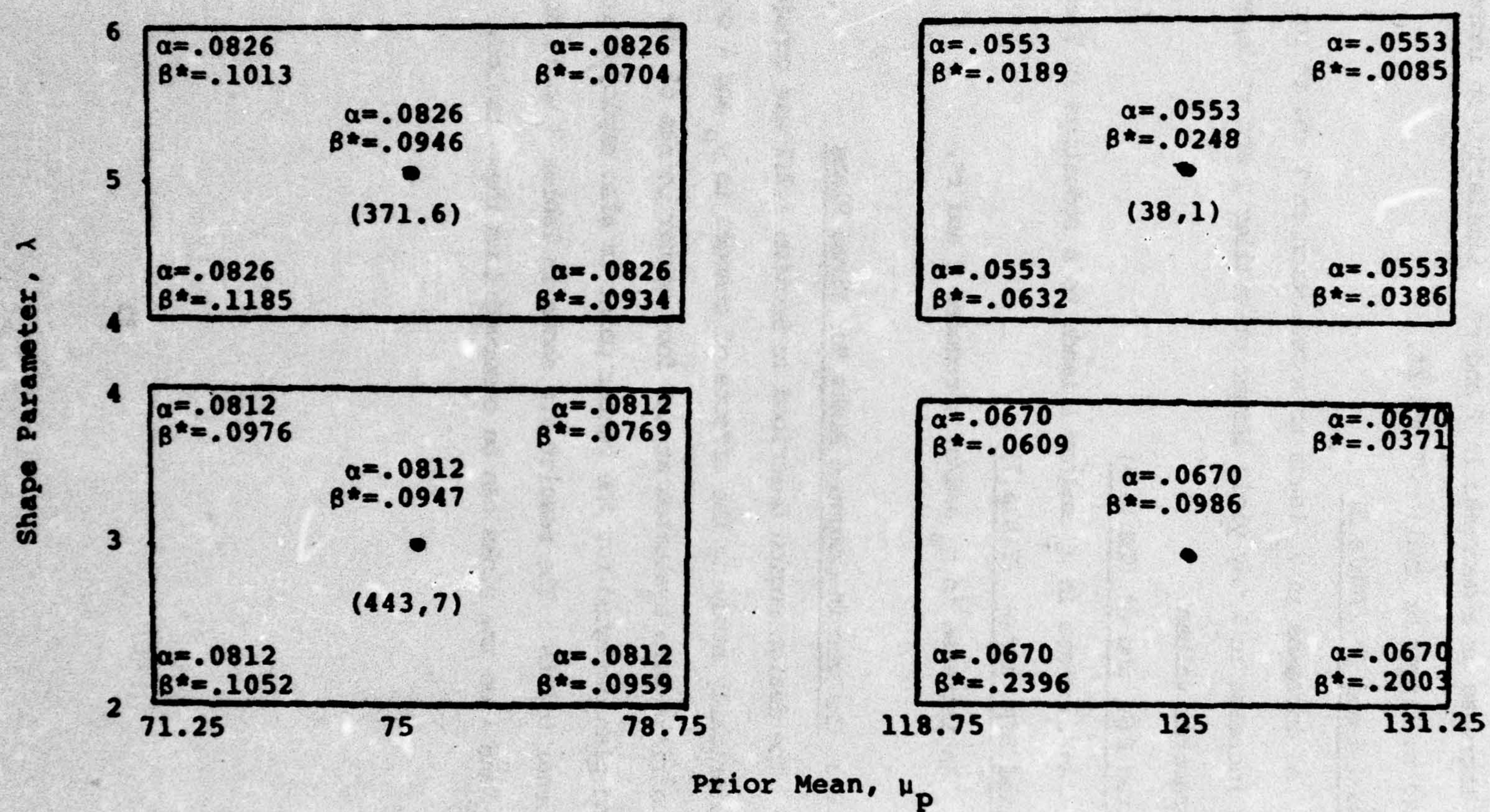


Table 9

Effect of Changes in μ_p and λ on Risks: Criterion $(\bar{\alpha},\bar{\beta})$

$(\theta_1=50,\ \theta_0=100)$

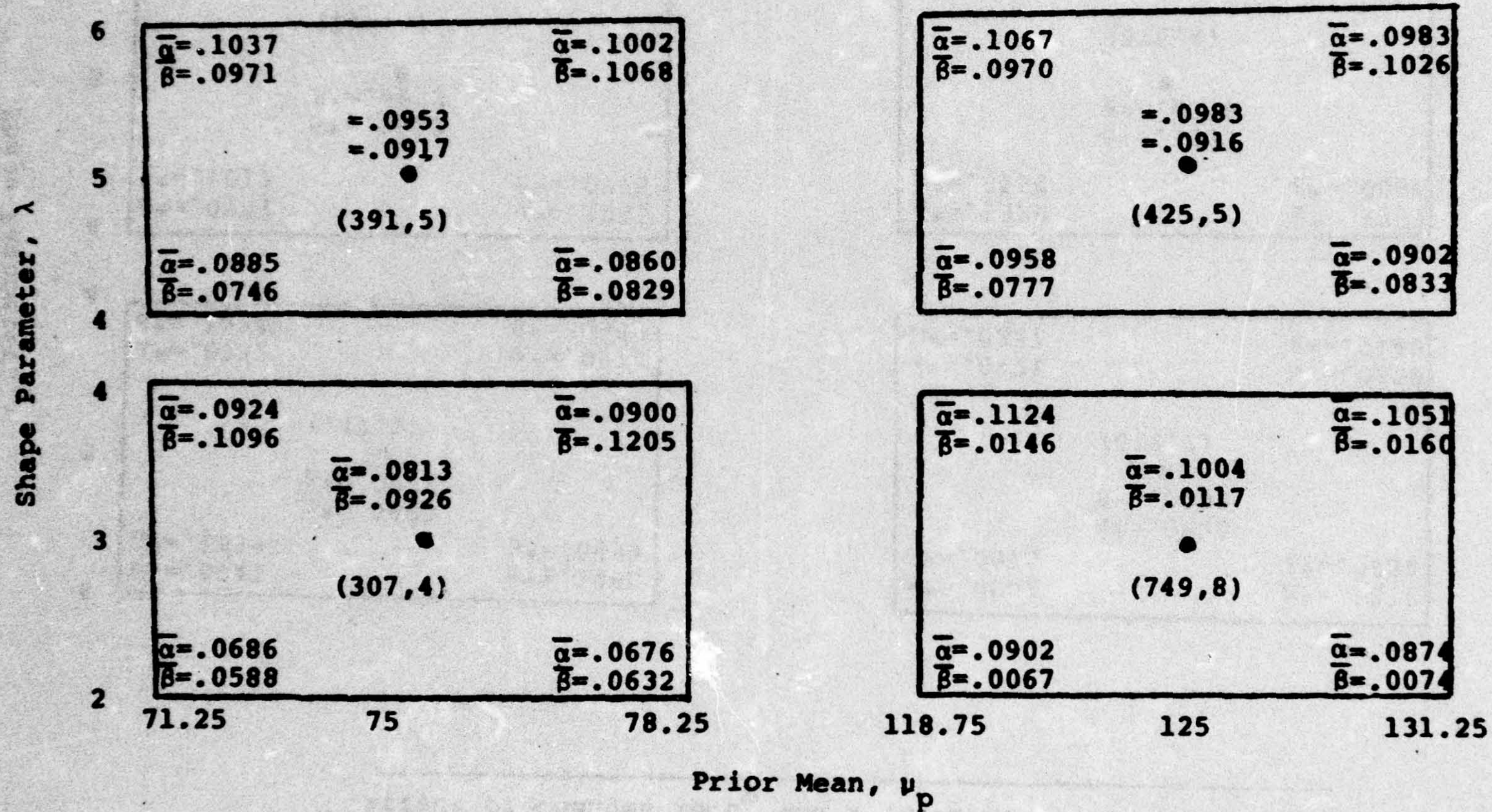


Table 10

Effect of Changes in μ_p and λ on Risks: Criterion (α^*, β^*)

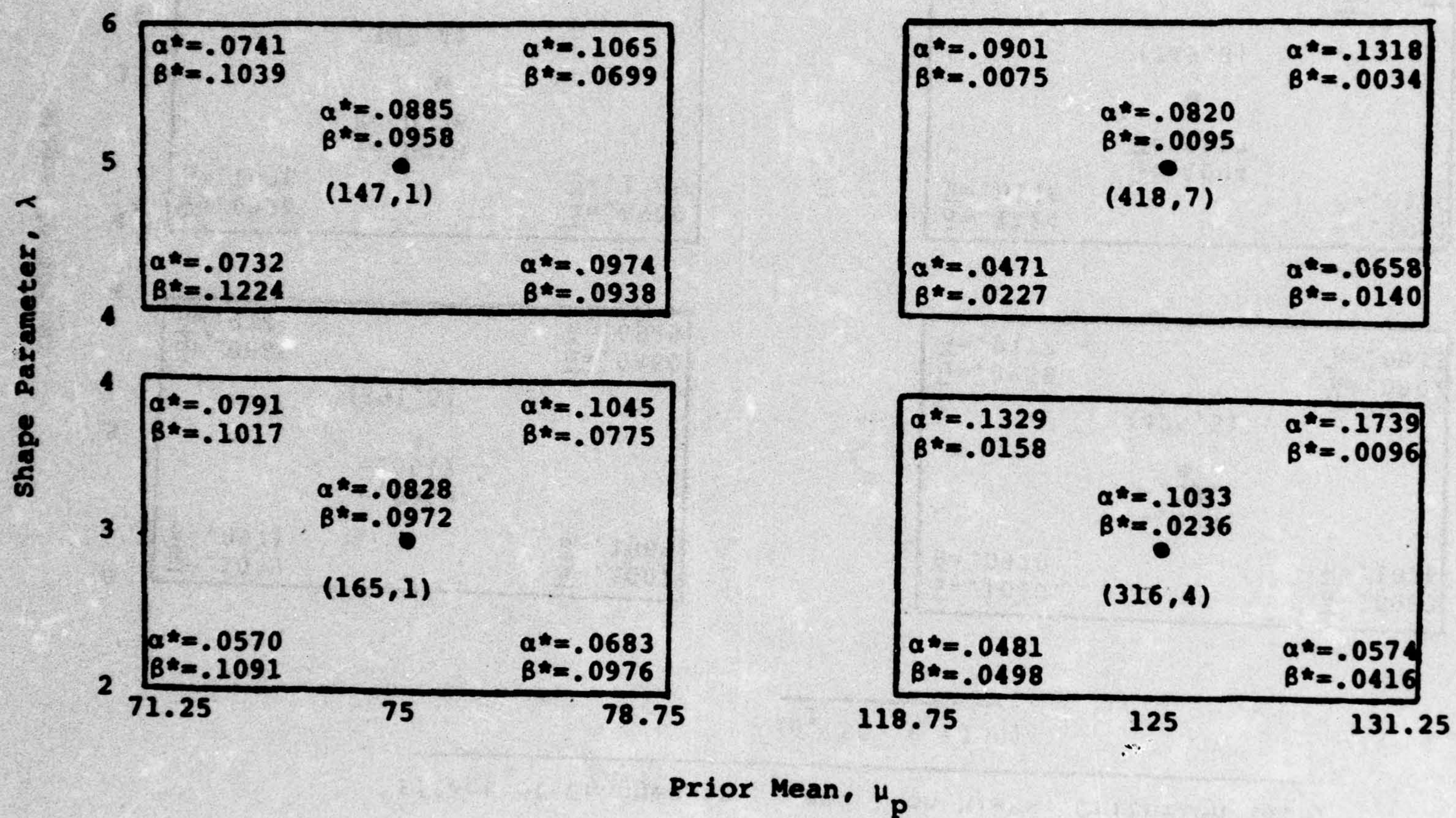


Table 11

Effect of Changes in μ_p and λ on Risks: Criterion $(\bar{\alpha},\beta^*)$

$(\theta_1=50,\ \theta_o=100)$

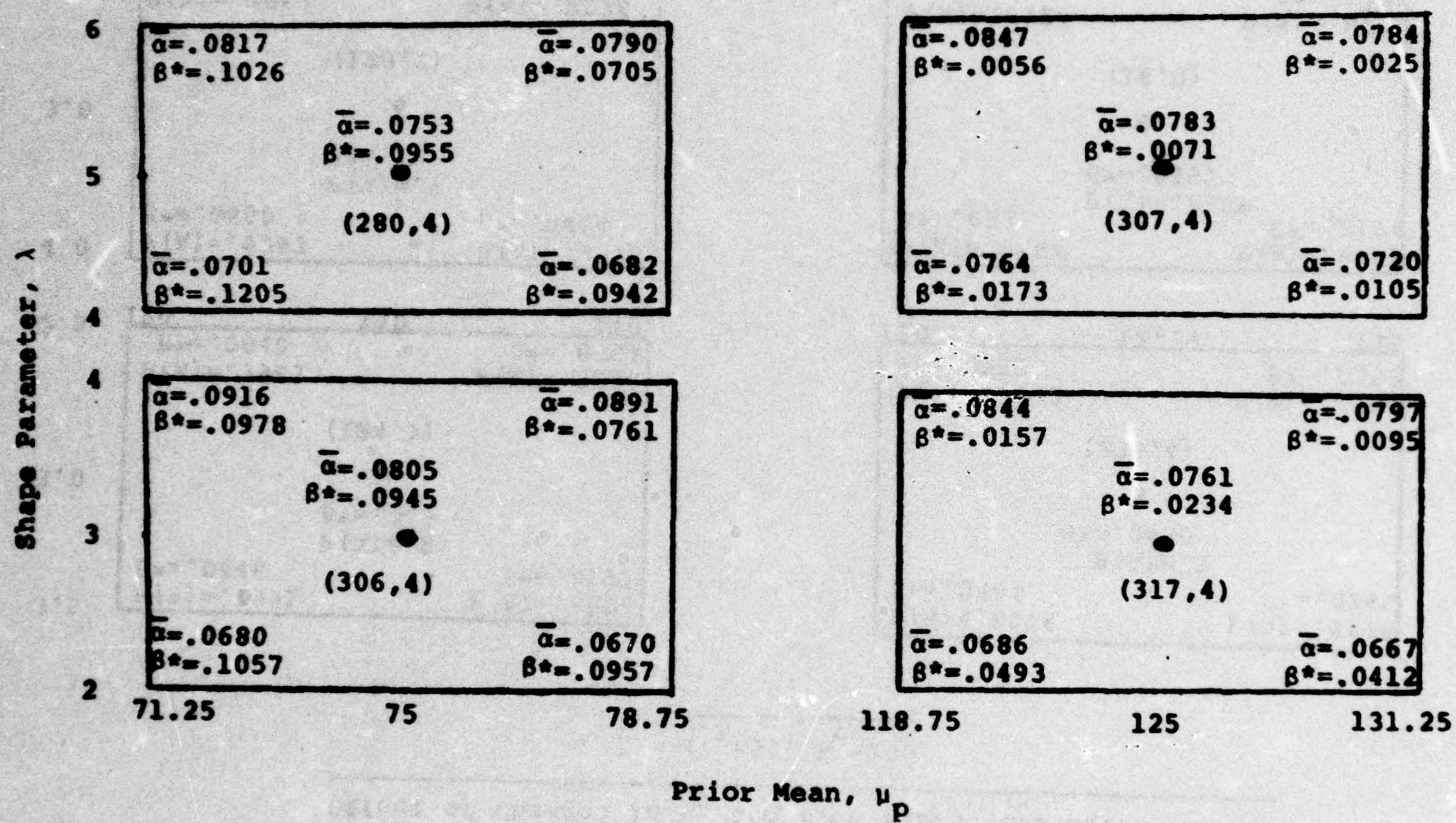


Table 12

Effect of Changes in μ_p and λ on Risks: Criterion $P(A), \beta^*$

($\theta_1=50$, $\theta_o=100$)

(184,3)

Shape Parameter, λ	120	130	140
3.5	P(A)=.7991 β*=.0446		P(A)=.8506 β*=.0227
3.0		P(A)=.8 β*=.045	
2.5	P(A)=.7963 β*=.0840		P(A)=.8226 β*=.0554

(238,4)

Shape Parameter, λ	120	127.5	135
3.5	P(A)=.8076 β*=.0404		P(A)=.8498 β*=.0245
3.0		P(A)=.8 β*=.0450	
2.5	P(A)=.7303 β*=.0752		P(A)=.7783 β*=.0553

(120,3)

Shape Parameter, λ	114	120	126
4.0	P(A)=.9203 β*=.0660		P(A)=.9376 β*=.0414
3.0		P(A)=.9 β*=.1	
2.0	P(A)=.7963 β*=.2168		P(A)=.8226 β*=.1841

(16,0)

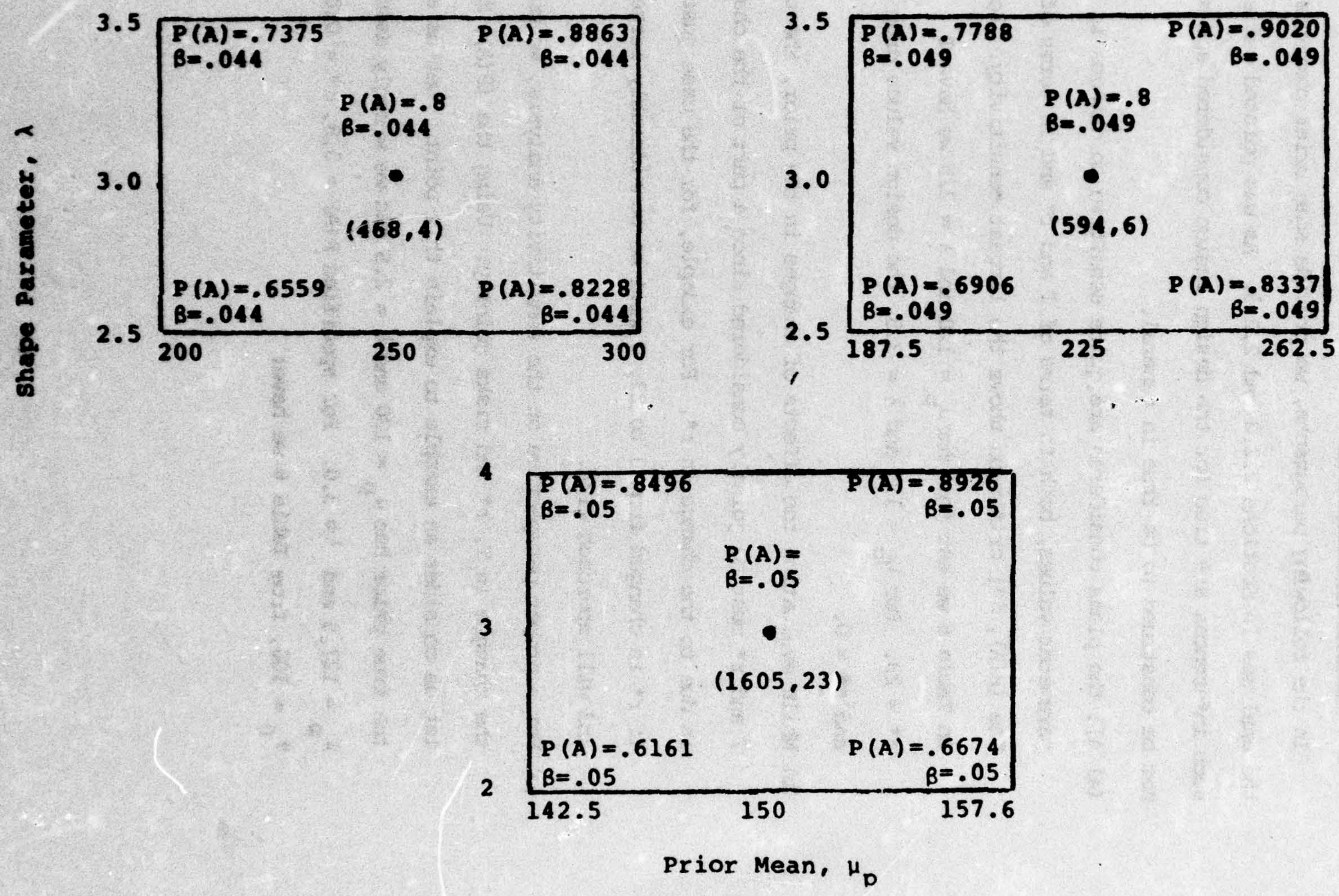
Shape Parameter, λ	142.5	150	157.5
4.0	P(A)=.8664 β*=.0235		P(A)=.8782 β*=.0124
3.0		P(A)=.8589 β*=.0497	
2.0	P(A)=.8124 β*=.1762		P(A)=.8279 β*=.1400

Prior Mean, μ_p

Table 13

Effect of Changes in μ_p and λ on Risks: Criterion $(P(A), \beta)$

$(\theta_1 = 50,\ \theta_o = 100)$



2.2.4 Some General Comments

In the following paragraphs, we provide some brief comments based on the analyses in Sections 2.2.2 and 2.2.3. As was pointed out earlier, such inferences are true for the design region considered and should not be construed to be true in general.

(a) All the plans considered are quite sensitive to changes in the prior parameter values, both in terms of T and r^* and in terms of the risks. The $(P(A), \beta^*)$ criterion shows the largest sensitivity. For example, in Table 6 we see that for $\mu_p = 120$ and $\lambda = 2.5$ we have $T = 1134$ and $r^* = 23$. For $\mu_p = 135$ and $\lambda = 3.5$, the design values drop to $T = 16$ and $r^* = 0$.

(b) While evaluating the effects of changes in the prior, the changes in T and r^* must be jointly considered since a part of the change in T is due to the change in r^*. For example, for the case just considered, if r^* is changed from 0 to 23, T will be considerably larger than 16 and will approach 1134.

(c) For a proper perspective on the sensitivity analysis, we must look at the changes in T, r^* and risks jointly. Using the $(P(A), \beta^*)$ criterion, let us consider an example to explain this point. Let us assume that the true prior has $\mu_p = 120$ and $\lambda = 2.5$ and we wrongly assume it to have $\mu_p = 127.5$ and $\lambda = 3.0$. For specified $P(A) = 0.8$, $\beta^* = 0.045$ and $\theta_0 = 100$, from Table 6 we have:

Assumed Prior: Designed Plan (238, 4)

True Prior: Designed Plan (1134, 23)

Thus, the error in the prior has led to reduced T and r*. Once the plan (238, 4) is implemented, the true risks will correspond to the true prior and will not equal 0.8 and 0.045.

From Table 12, we have the corresponding risks as:

Assumed Prior: $P(A) = 0.8$, $\beta^* = 0.045$

True Risks: $P(A) = 0.7303$, $\beta^* = 0.0752$

This example shows that the error on the prior has led to a reduced T and r* and higher risks. Similarly it may happen that T and r* get larger and the true risks get smaller.

It, therefore, follows that if we consider the cost of testing and the cost of making wrong decisions, an error in the prior may lead to increased testing cost and reduced cost due to risks or vice versa. This balancing of costs indicates that the cost may be less sensitive to changes in the prior and points out the need to consider the cost structure in the design of plans.

2.3 Test Times and Test Time Relationships

The effect of varying μ_p and varying λ on T is shown on Figures 7 and 8 respectively for several different criteria. The slopes of these curves are a measure of the sensitivity of T to changes in μ_p and λ. A comparison of test times based on some numerical values of risks is somewhat meaningless since the design criteria are different. The plots in Figures 7 and 8 are provided only for the sake of completeness.

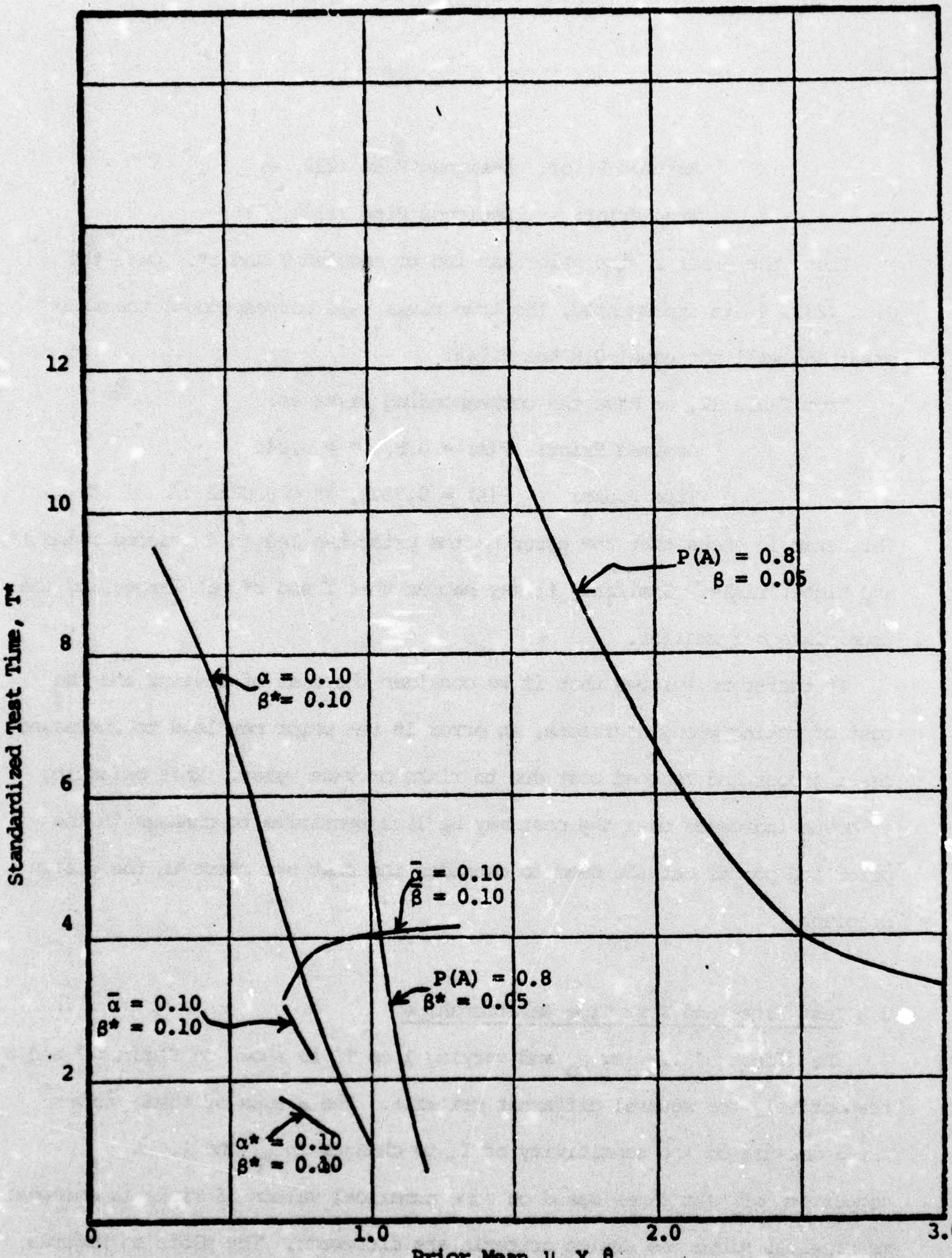


Figure 7: Effect of μ_p on T* (K = 2, λ = 4)

NOTE: Discontinuities exist on all curves; curves are drawn only for information.

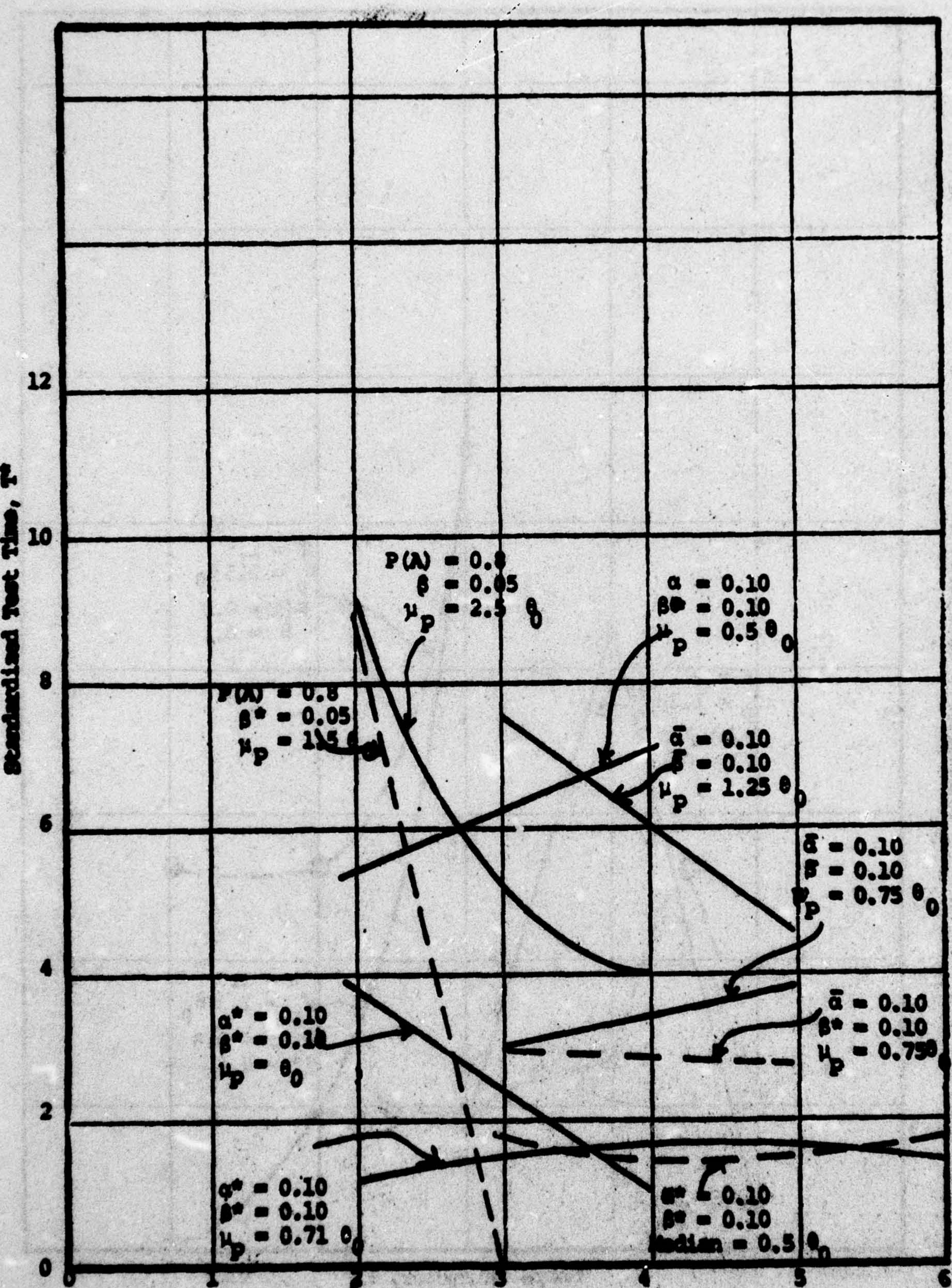


Figure 8 (a) Effect of λ on T* (K = 2)

NOTE: Discontinuities exist on all curves; curves are drawn only for information.

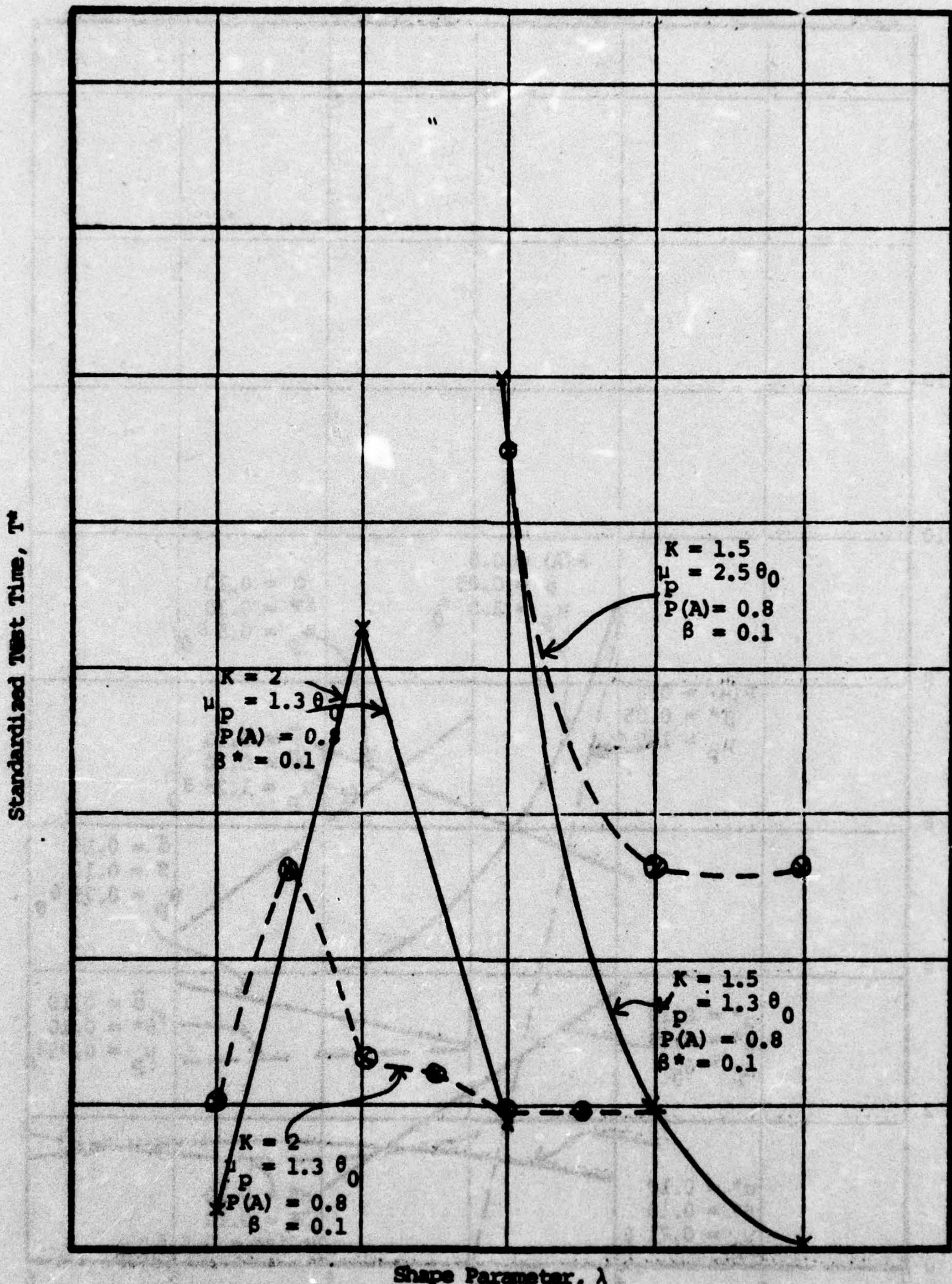


Figure 8 (b) Effect of λ on T*

NOTE: Discontinuities exist on all curves; curves are drawn only for information.

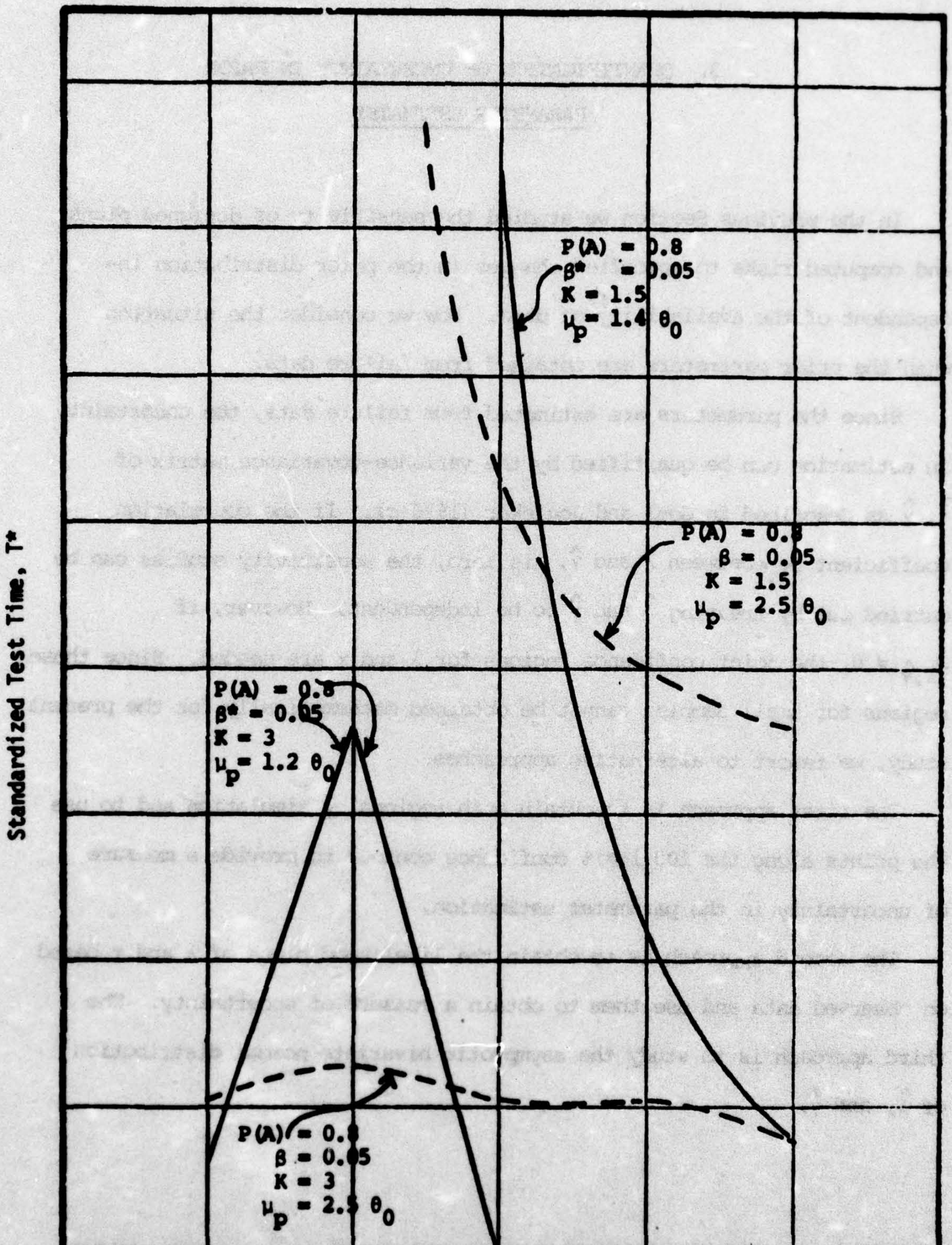


Figure 8(c) Effect of λ on T^*

NOTE: Discontinuities exist on all curves; curves are drawn only for information.

3. QUANTIFICATION OF UNCERTAINTY IN PRIOR PARAMETER ESTIMATES

In the previous Section we studied the sensitivity of designed plans and computed risks to specified changes in the prior distribution independent of the availability of data. Now we consider the situation when the prior parameters are obtained from failure data.

Since the parameters are estimated from failure data, the uncertainty in estimation can be quantified by the variance-covariance matrix of $\hat{\lambda}$, $\hat{\gamma}$ as described in Goel and Joglekar (1976 c). If the correlation coefficient $\rho_{\hat{\lambda},\hat{\gamma}}$ between $\hat{\lambda}$ and $\hat{\gamma}$, is zero, the sensitivity studies can be carried out by treating $\hat{\lambda}$ and $\hat{\gamma}$ to be independent. However, if $\rho_{\hat{\lambda},\hat{\gamma}} \neq 0$, the joint confidence regions for λ and γ are needed. Since these regions for small samples cannot be obtained mathematically for the present study, we resort to alternative approaches.

The first approach is to obtain such regions by simulation and to use the points along the $100(1-\alpha)\%$ confidence contour to provide a measure of uncertainty in the parameter estimation.

The second approach is to obtain the likelihood plots of λ and γ based on observed data and use them to obtain a measure of uncertainty. The third approach is to study the asymptotic bivariate normal distribution of $\hat{\lambda}$, and $\hat{\gamma}$.

In the following subsections we use these three approaches for sensitivity studies. Two sensitivity analyses are conducted. First the effect of the uncertainty in the prior parameter estimates on the normalized test time T^* ($= T/\theta_0$) and acceptance number r^* for a single sample truncated plan for a system is determined. Four risk criteria, namely $(\bar{\alpha},\bar{\beta})$, $(\bar{\alpha},\beta^*)$, (α^*,β^*) and $(P(R),\beta^*)$ (see Goel and Joglekar 1976(b) for details) are considered. In each case designed (T^*,r^*) values are obtained for specified risk values and estimated $(\hat{\lambda},\hat{\gamma})$. Then, keeping the risks constant, designed values are obtained for some selected values of λ and γ using one of the above-mentioned three approaches. The variation in the designed (T^*,r^*) values provides a measure of sensitivity.

A second sensitivity analysis is conducted to determine the effect of uncertainty associated with $\hat{\lambda}$ and $\hat{\gamma}$ on the risks. The four risk criteria given above are considered. In each case, designed T^* and r^* values corresponding to $\hat{\lambda}$ and $\hat{\gamma}$ are used to compute appropriate risks at selected (λ,γ) points. The variation in risks constitutes a measure of sensitivity.

The reasons behind the observed sensitivity and the choice of criteria on the basis of sensitivity are discussed next. An approach to determine the amount of data necessary to obtain a prespecified sensitivity is outlined. Also, a comparison of the results from the simulation, likelihood and bivariate normal approaches provides practical guidelines for the quantification of uncertainty.

3.1 Variance-Covariance Matrix for λ, γ

Parameter estimation has been considered in detail in Goel and Joglekar (1976 c). It has been shown that if

$$f(t|\theta) = \frac{1}{\theta} e^{-t/\theta} \quad ; \quad t \geq 0, \ \theta > 0 \tag{3}$$

and

$$g(\theta) = \frac{\gamma^{\lambda}}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} \quad ; \quad \theta, \gamma, \lambda > 0 \tag{4}$$

then the marginal density of the number of failures X in a fixed time T is negative binomial i.e.

$$f(x) = \frac{(\lambda+x-1)!}{x!(\lambda-1)!} \left(\frac{T}{T+\gamma}\right)^x \left(\frac{\gamma}{T+\gamma}\right)^{\lambda}; \quad x = 0,1,2,3\ldots \tag{5}$$

The converse is also true. If the observed frequency n_x corresponding to x = 0 is not recorded then X has a truncated negative binomial distribution given by

$$f(x) = \frac{(\lambda+x-1)!}{x!(\lambda-1)!\{1-(\frac{\gamma}{T+\gamma})^{\lambda}\}} \cdot \left(\frac{T}{T+\gamma}\right)^x \left(\frac{\gamma}{T+\gamma}\right)^{\lambda}; \ x = 1,2.. \tag{6}$$

It should be noted that the prior parameters λ and γ also form the parameters of the distribution of the observable random variable X. The parameters can be estimated by fitting a negative binomial or a truncated negative binomial distribution to the observed values of X depending on whether observations corresponding to x = 0 are recorded or not. In our case observations for x = 0 are missing and hence the maximum likelihood

estimates for the truncated negative binomial case are obtained. In this case the log likelihood is

$$\ell nL = \sum_{x=1}^{k} n_x\{\ell n(\lambda+x-1)! - \ell n(x)! - \ell n(\lambda-1)! + \ell n(\frac{\gamma}{T+\gamma})$$

$$+ x\ell n(\frac{T}{T+\gamma}) - \ell n[1-(\frac{\gamma}{T+\gamma})^{\lambda}]\} \tag{7}$$

Estimates $\hat{\lambda}$ and $\hat{\gamma}$ can be obtained by an iterative solution of $\ell nL/\partial\lambda = 0$ and $\ell nL/\partial\gamma = 0$. The variance-covariance matrix for $\hat{\lambda}$ and $\hat{\gamma}$ can be numerically obtained by taking the second partial derivatives of ℓnL and is given by,

$$\begin{bmatrix} var(\hat{\gamma}) & cov(\hat{\gamma},\hat{\lambda}) \\ cov(\hat{\gamma},\hat{\lambda}) & var(\hat{\lambda}) \end{bmatrix} = -\begin{bmatrix} E\frac{\partial^2 \ell nL}{\partial\gamma^2} & E\frac{\partial^2 \ell nL}{\partial\gamma\partial\lambda} \\ E\frac{\partial^2 \ell nL}{\partial\lambda\partial\gamma} & E\frac{\partial^2 \ell nL}{\partial\lambda^2} \end{bmatrix}^{-1}_{(\gamma=\hat{\gamma},\ \lambda=\hat{\lambda})} \tag{8}$$

and the correlation coefficient is given by

$$\rho_{\hat{\gamma},\hat{\lambda}} = \frac{Cov(\hat{\gamma},\hat{\lambda})}{\{Var(\hat{\gamma})\ Var(\hat{\lambda})\}^{1/2}} \tag{9}$$

For the present purpose, we reconsider three of the eight data sets of Schafer (1970) analyzed in Goel and Joglekar (1976 c). The data is given in Table 14 where x denotes the number of failures in 4320 hours of

TABLE 14

OBSERVED FAILURE DATA*

Transmitter		Search Indicator IP-128A		Search MVPS PP-3132	
x	n_x	x	n_x	x	n_x
1	16	1	21	1	11
2	10	2	8	2	11
3	7	3	8	3	10
4	5	4	2	4	7
5	3	5	5	5	5
6	3	6	4	6	4
7	1	7	3	7	1
8	1	8	2	8	4
9	1	9	1	9	7
12	1	19	1	10	3
15	1			13	1
33	1			14	3
				15	1
				16	3
				19	1
				22	1
				44	1
	50		55		74

* Data from Schafer (1970)

testing and n_x is the observed frequency. Since observations corresponding to $x = 0$ are missing, a truncated negative binomial is fitted and the maximum likelihood estimates, their variances and the correlation coefficient are:

System	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\sigma}_{\hat{\lambda}}$	$\hat{\sigma}_{\hat{\gamma}}$	$\hat{\rho}_{\hat{\lambda},\hat{\gamma}}$
Transmitter	0.2054	680.0	0.28	318.0	0.86
Search Indicator	0.4407	1152.0	0.40	542.0	0.90
Search MVPS	0.8646	714.0	0.31	211.0	0.89

Since $\hat{\rho}_{\hat{\gamma},\hat{\lambda}}$ is approximately 0.9 for all three cases, the individual variance estimates do not provide sufficient quantification of uncertainty and joint confidence regions for λ and γ must be obtained. As mentioned earlier, explicit expression for the joint density does not seem feasible since the parameter estimates themselves are not explicitly known.

3.2 Simulation Study

A simulation study was conducted to obtain the joint and the marginal densities of λ and γ. The study can best be explained by an example. Consider the failure data in Table 1 for the search indicator. Based upon 55 failures the estimated parameters are $\hat{\lambda} = 0.4407$ and $\hat{\gamma} = 1152.0$. The steps in the simulation study are as follows:

1. Take $\lambda = 0.4407$ and $\gamma = 1152.0$ and generate 55 observations from a truncated negative binomial distribution with parameters λ, γ.
2. Estimate the parameters λ and γ for the generated failure data using maximum likelihood estimation procedure.
3. Repeat the above steps N (say 200) times. A plot of $\hat{\lambda}$ vs $\hat{\gamma}$ gives the simulated joint density. The histograms for $\hat{\lambda}$ and $\hat{\gamma}$ constitute the simulated *marginal densities*.

Using the above procedure, the following four simulationsswere conducted.

System	λ	γ	No. of Observations for each simulation	No. of Simulations
Transmitter	0.2054	680.0	50	200
Search Indicator	0.4407	1152.0	55	200
Search MVPS	0.8646	714.0	74	200
Search Indicator	0.4407	1152.0	55	1000

Based on the above simulations, four sets of $\hat{\lambda}$ and $\hat{\gamma}$ values are obtained, the first three consisting of 200 pairs of values and the fourth one with 1000 pairs of $\hat{\lambda},\hat{\gamma}$. The first three sets of $\hat{\lambda},\hat{\gamma}$ values are given in Appendix A, Tables A.1 to A.3. The simulated joint densities and the simulated marginal densities are obtained from these sets of $(\hat{\lambda},\hat{\gamma})$ values. The joint densities for cases (a) through (c) are given in Figures 9 to 11 respectively, and the marginal densities for $\hat{\lambda}$ and $\hat{\gamma}$ for the four cases are plotted in Figures 12 to 19. The computed values of sample mean, sample variance, skewness α_3 and kurtosis α_4 are also given in Figures 12 to 19.

The joint density of $\hat{\lambda}$ and $\hat{\gamma}$ has the interpretation that the 95% contour corresponds to the 95% confidence region for λ and γ. Points along the 95% confidence contour can be judiciously selected to quantify the uncertainty associated with the prior parameters for the purpose of sensitivity analysis.

The following observations may be made from the above simulation results.

(i) Some of the estimated parameter values are negative and are inadmissible. These negative values may be caused because of the nature of the simulated data in these few cases. This is not surprising when an iterative procedure is used to get the maximum likelihood estimates, especially when the value of λ is so close to zero. One commonly used approach is to take the negative value to be slightly greater than zero. Another possibility is to use Brass's modified moment estimates. These approximations are permissible only because it is known that the data is from a TNBD.

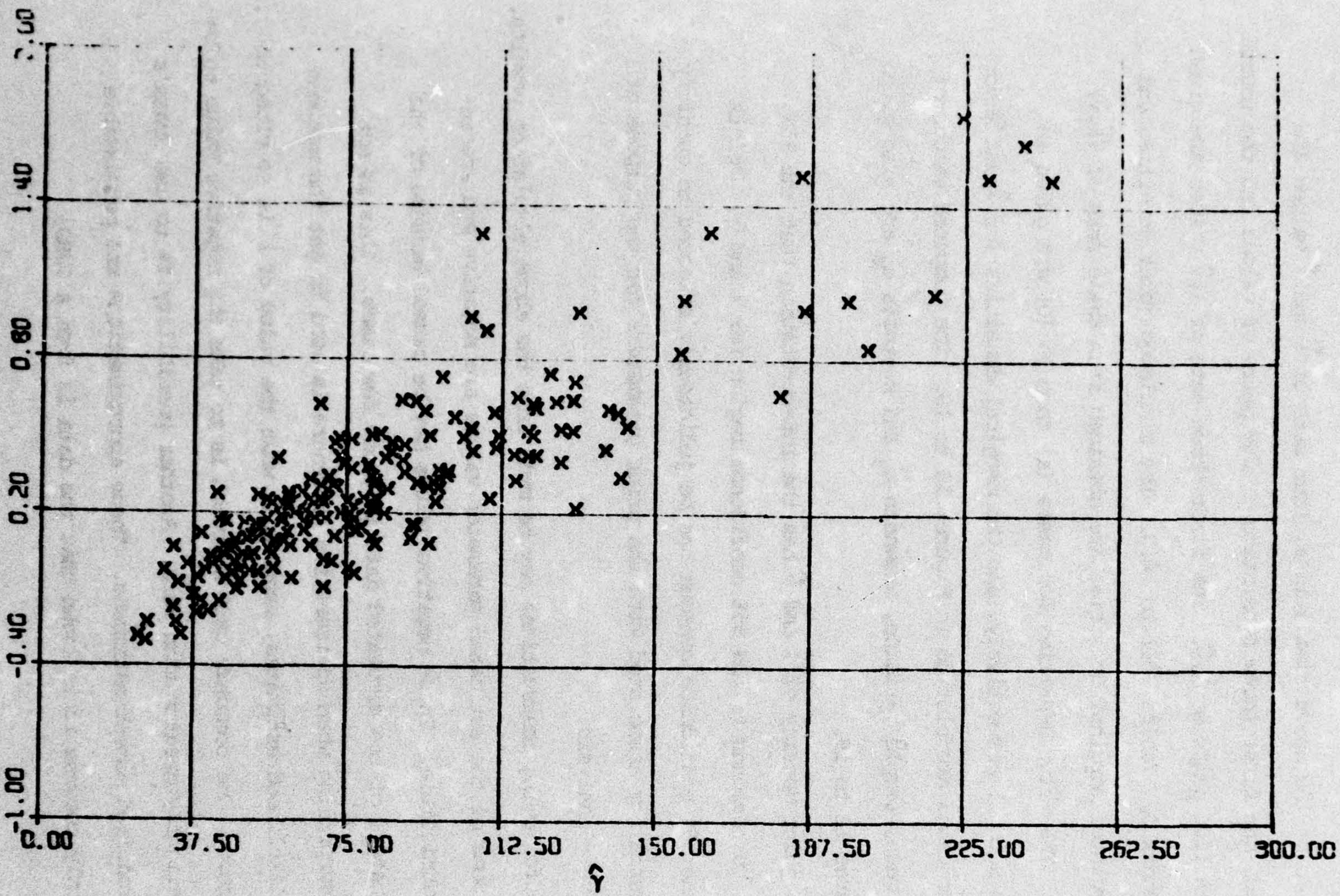


FIGURE 9. Plot of Estimated $(\hat{\gamma},\hat{\lambda})$ Values based on 200 Simulations

(sample size 50, γ = 680, λ = 0.2054)

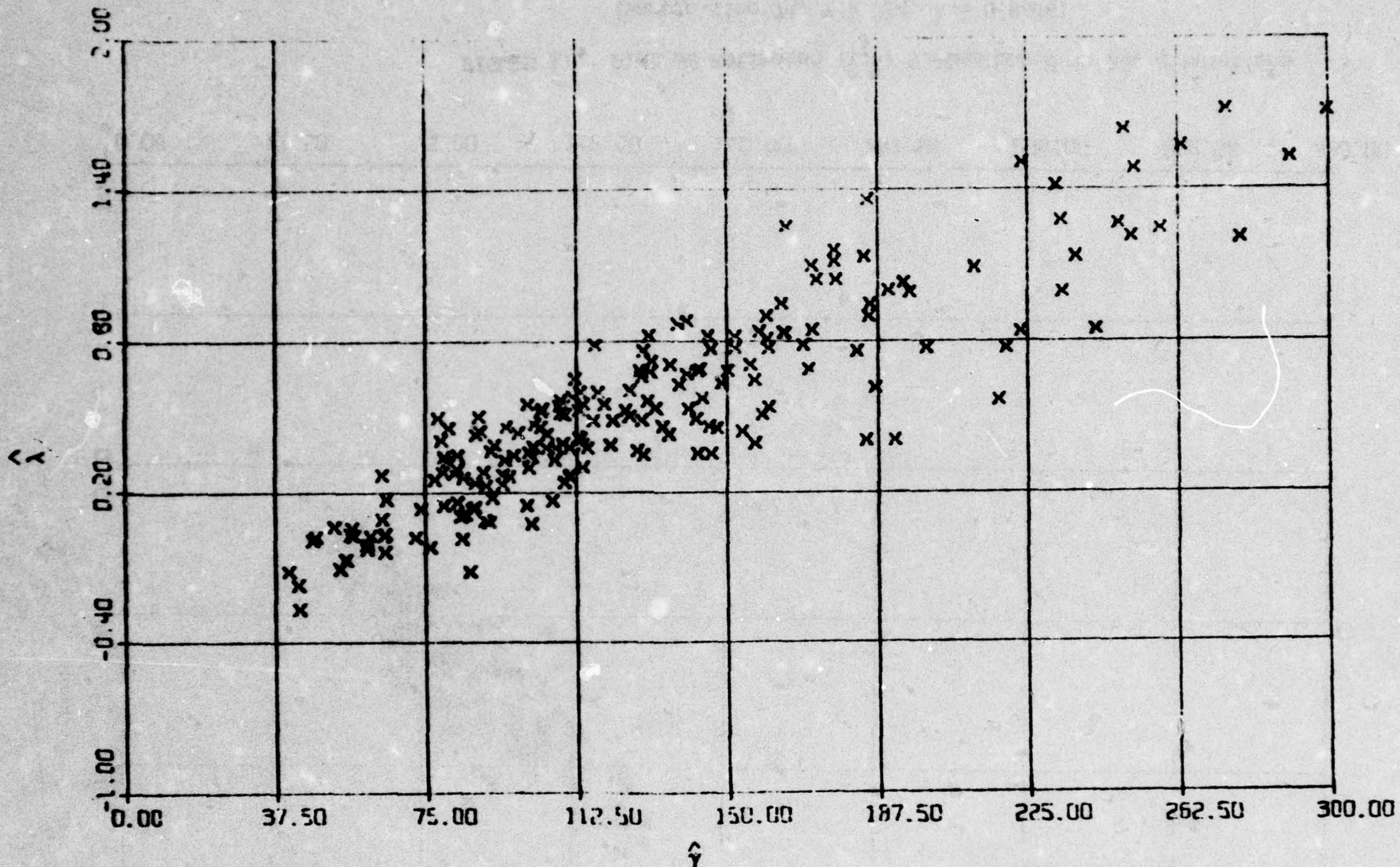


FIGURE 10. Plot of Estimated $(\hat{\gamma},\hat{\lambda})$ Values based on 200 Simulations

(Sample size 55, γ = 1152, λ = 0.4407)

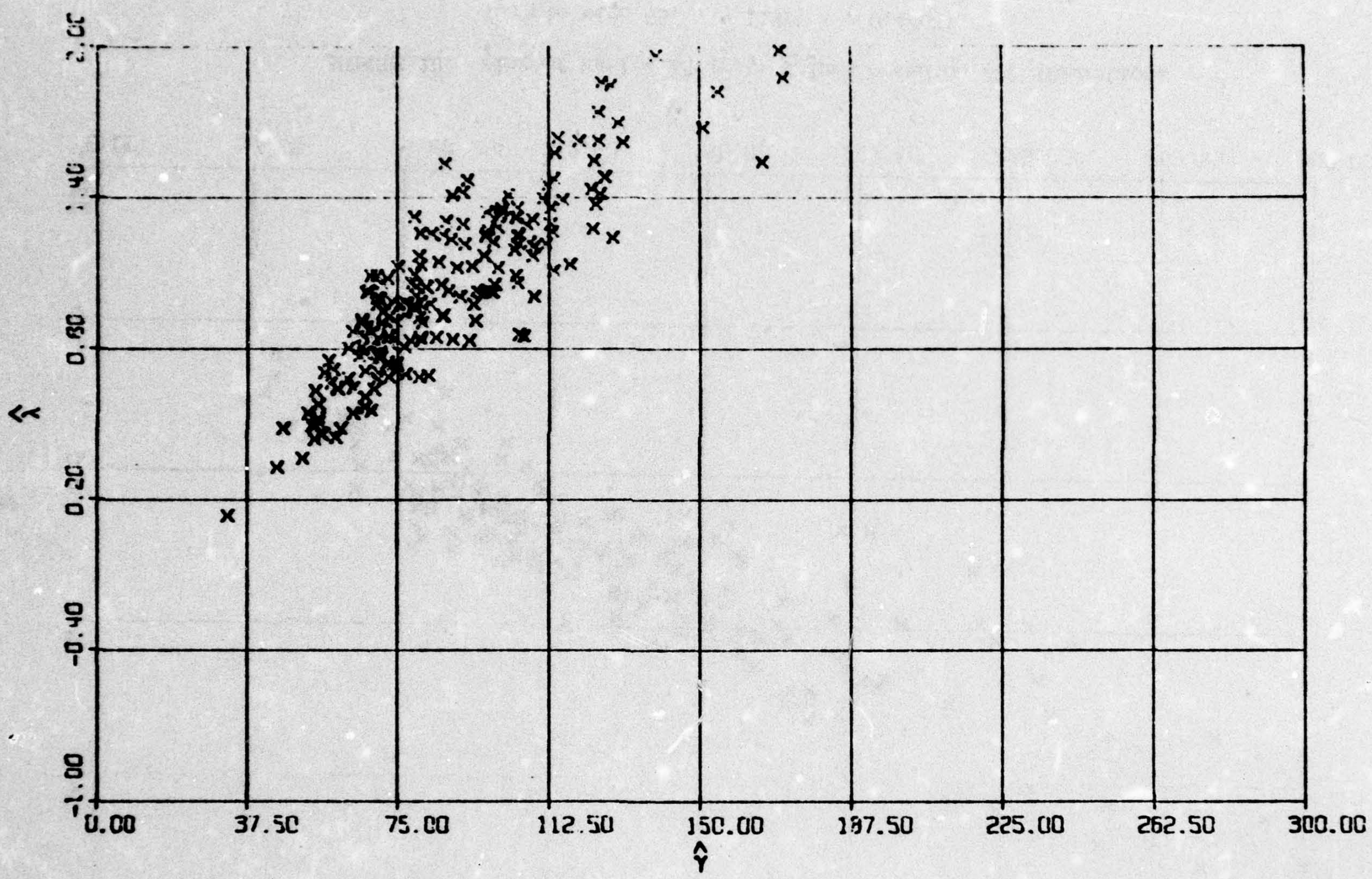


FIGURE 11. Plot of Estimated $(\hat{\gamma},\hat{\lambda})$ Values based on 200 Simulations

(Sample size 74, γ = 714, λ = 0.8646)

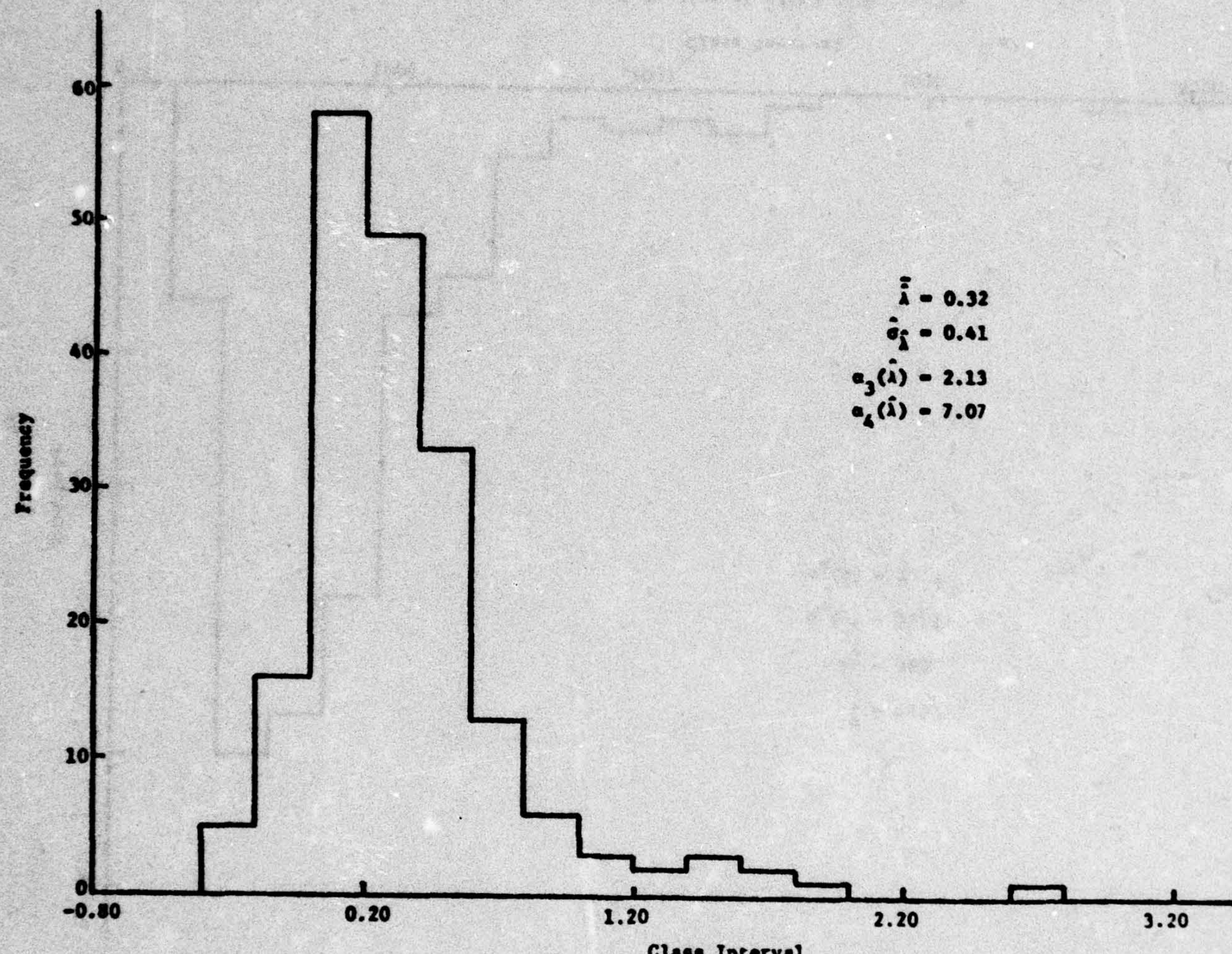


Figure 12. Marginal Density of $\hat{\lambda}(\lambda=0.2054, \gamma=680)$

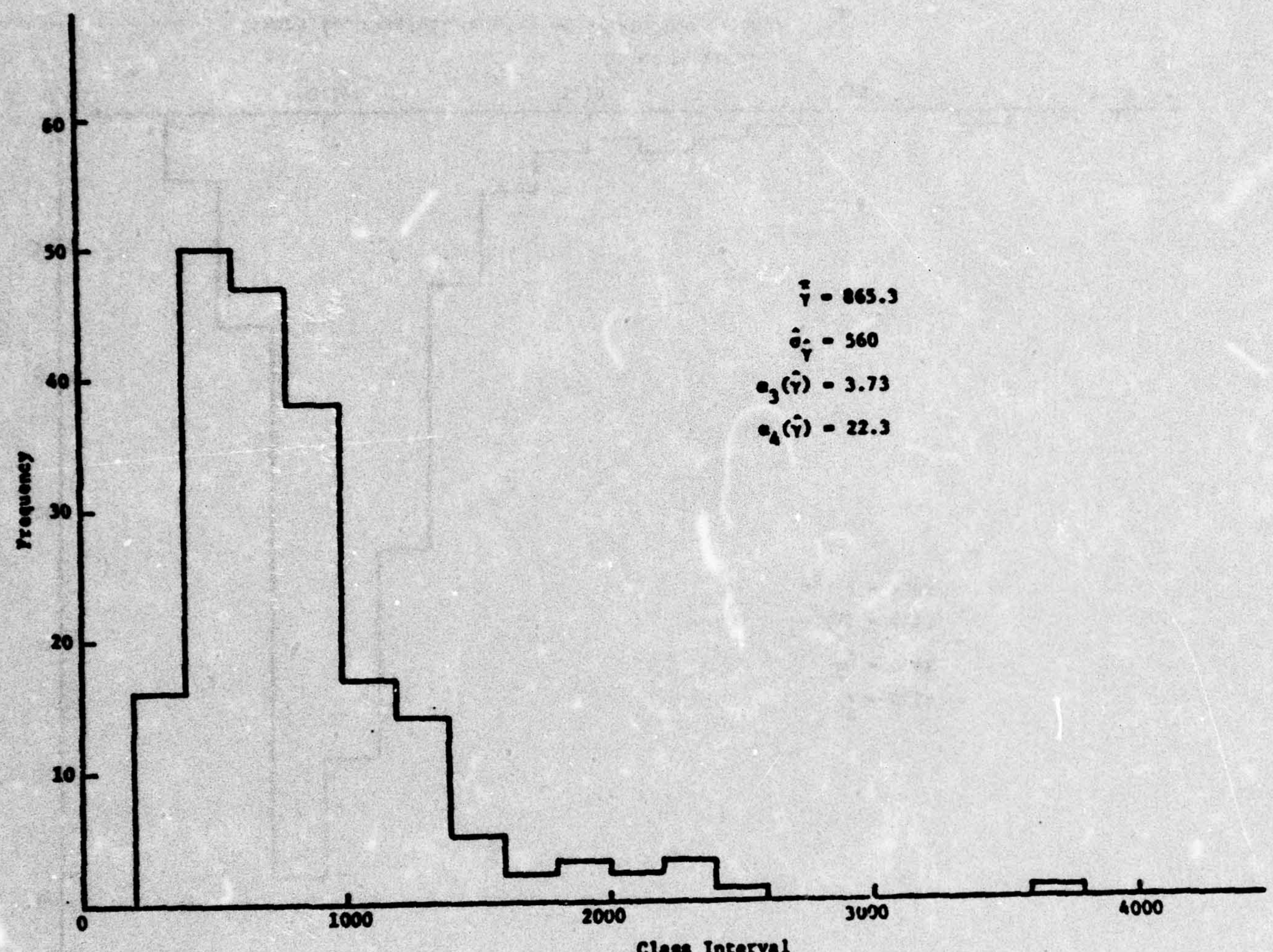


Figure 13. Marginal Density of $\hat{\gamma}(\lambda=0.2954,\ \gamma=680)$

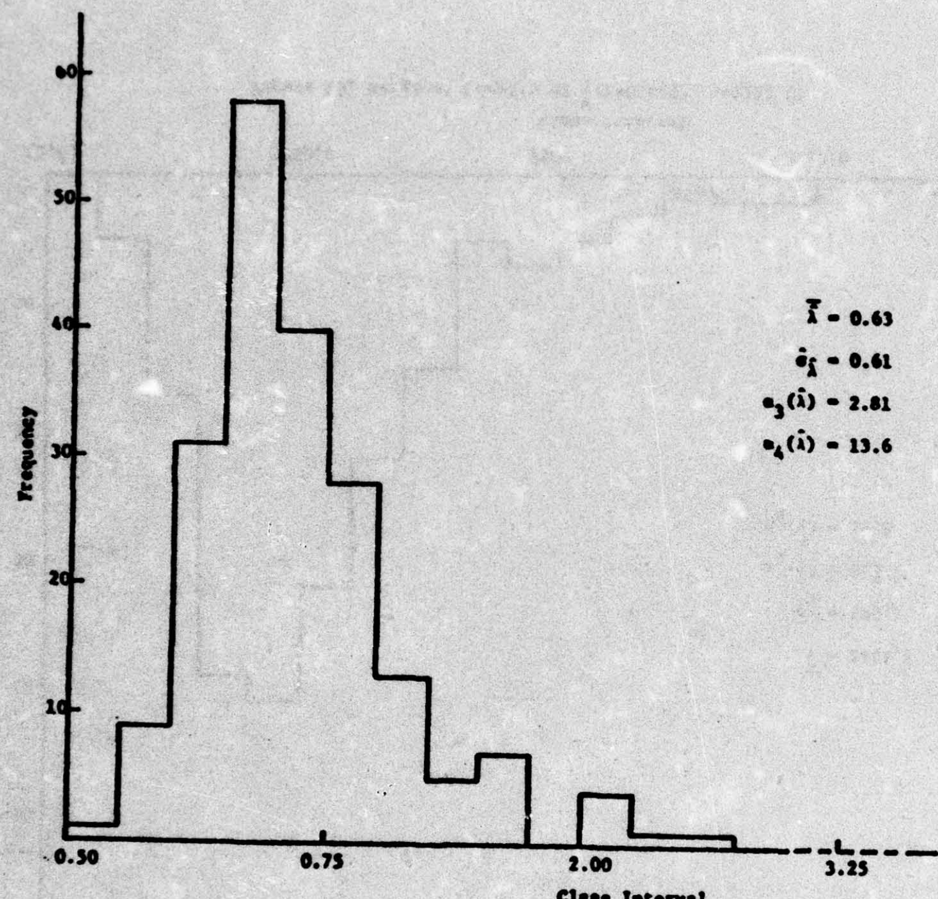


Figure 14. Marginal Density of $\hat{\lambda}$(λ= 0.4407, γ= 1152.0)

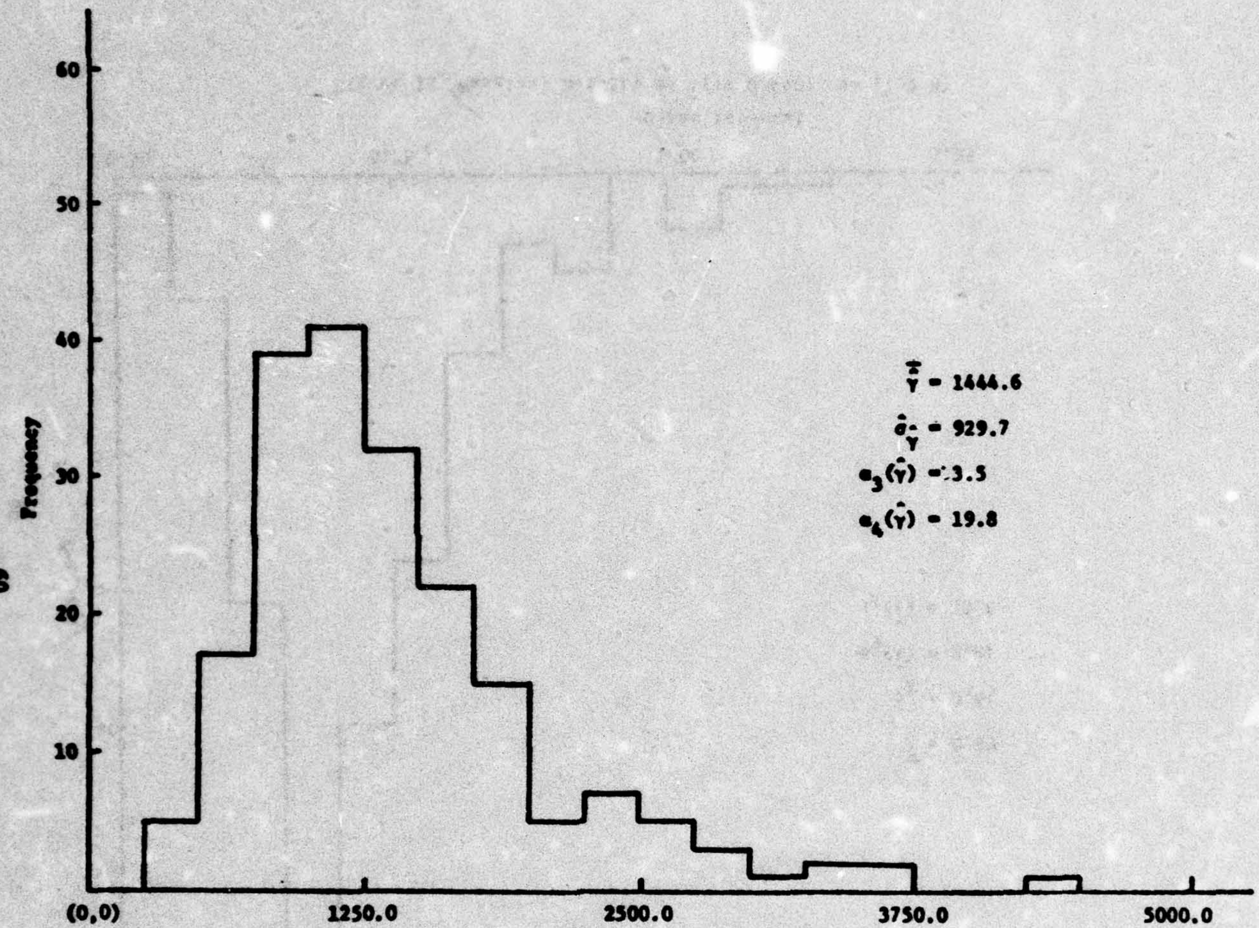


Figure 15, Marginal Density of $\hat{Y}$(Y=0.4407, γ=1152.0)

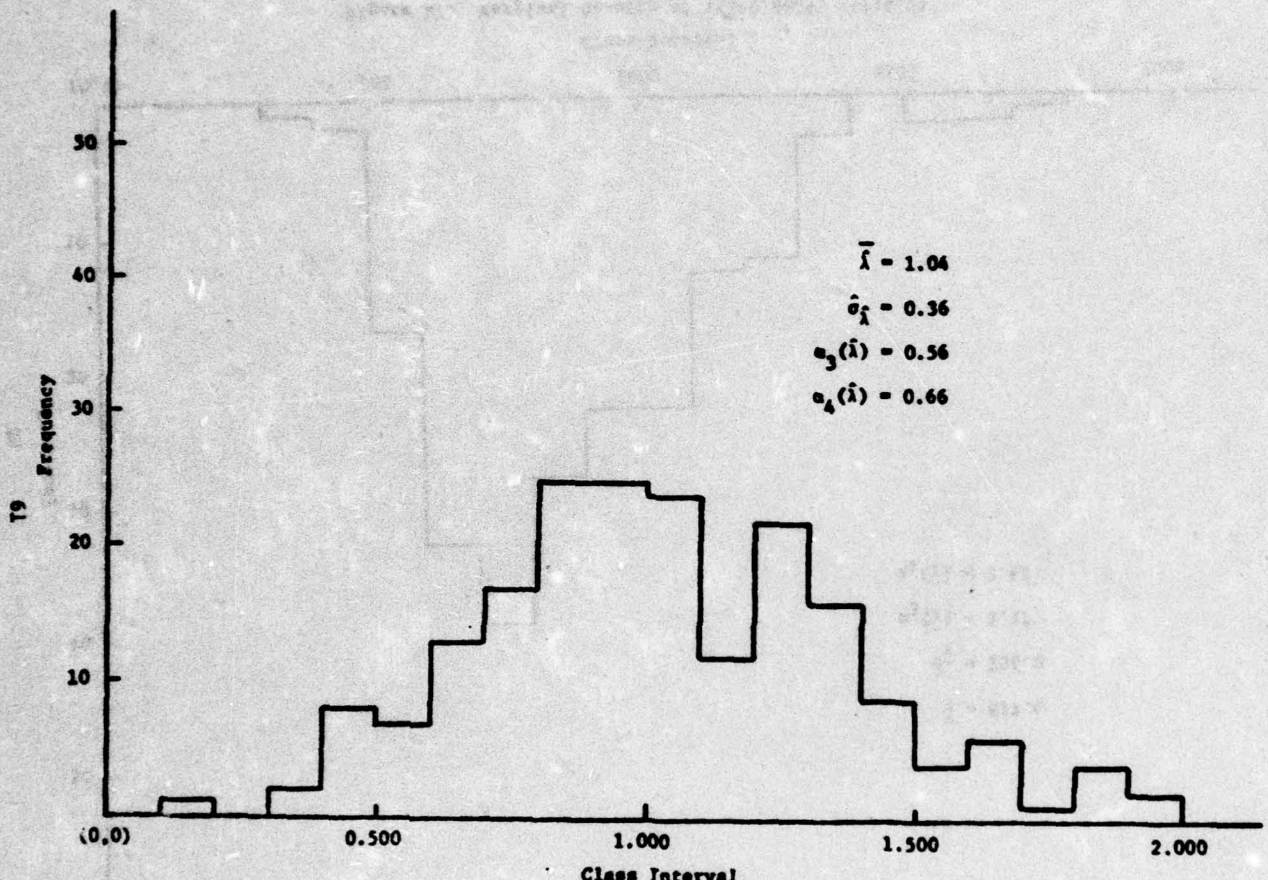


Figure 16. Marginal Density of $\hat{\lambda}(\lambda=0.8626,\ \gamma=714.0)$

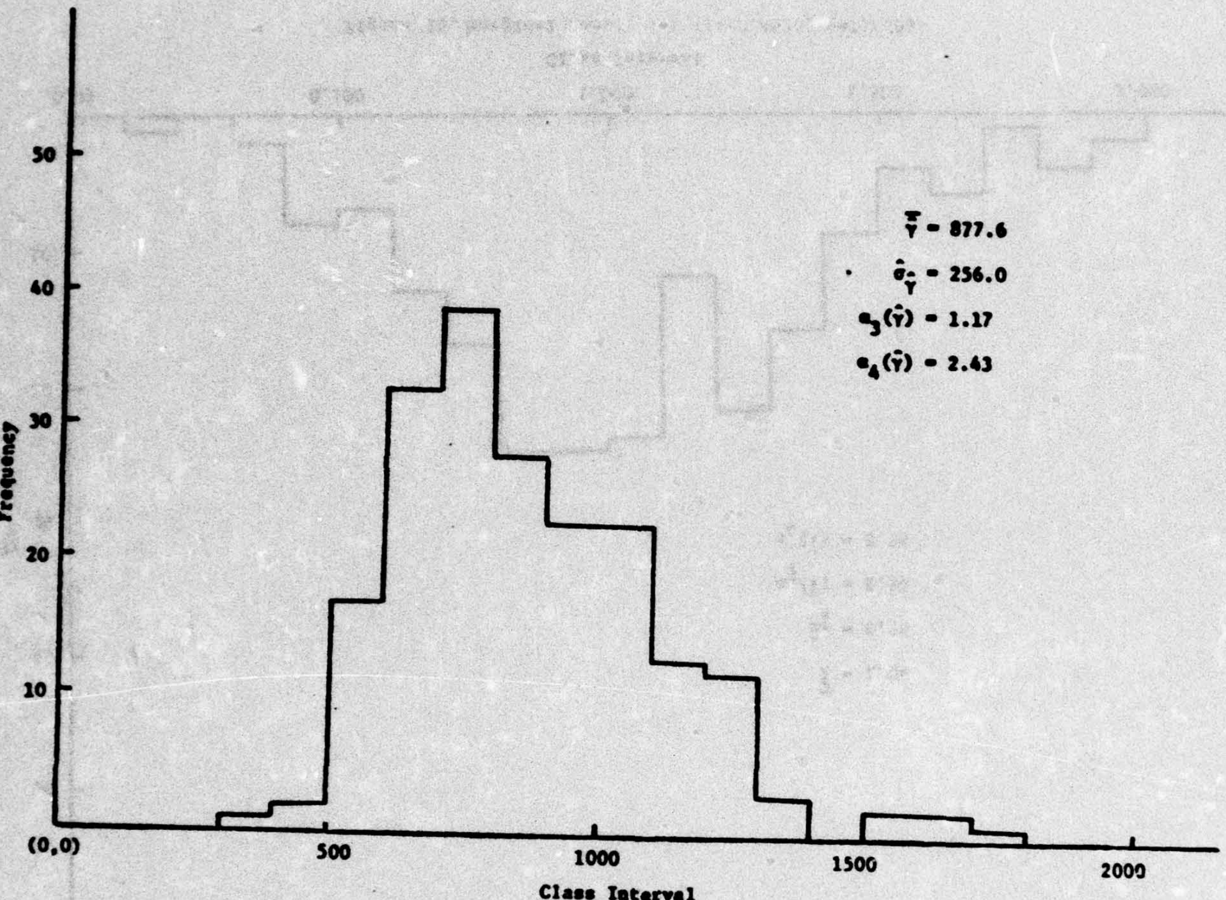


Figure 17. Marginal Density of $\hat{\gamma}$(λ=0.8626, γ=714.0)

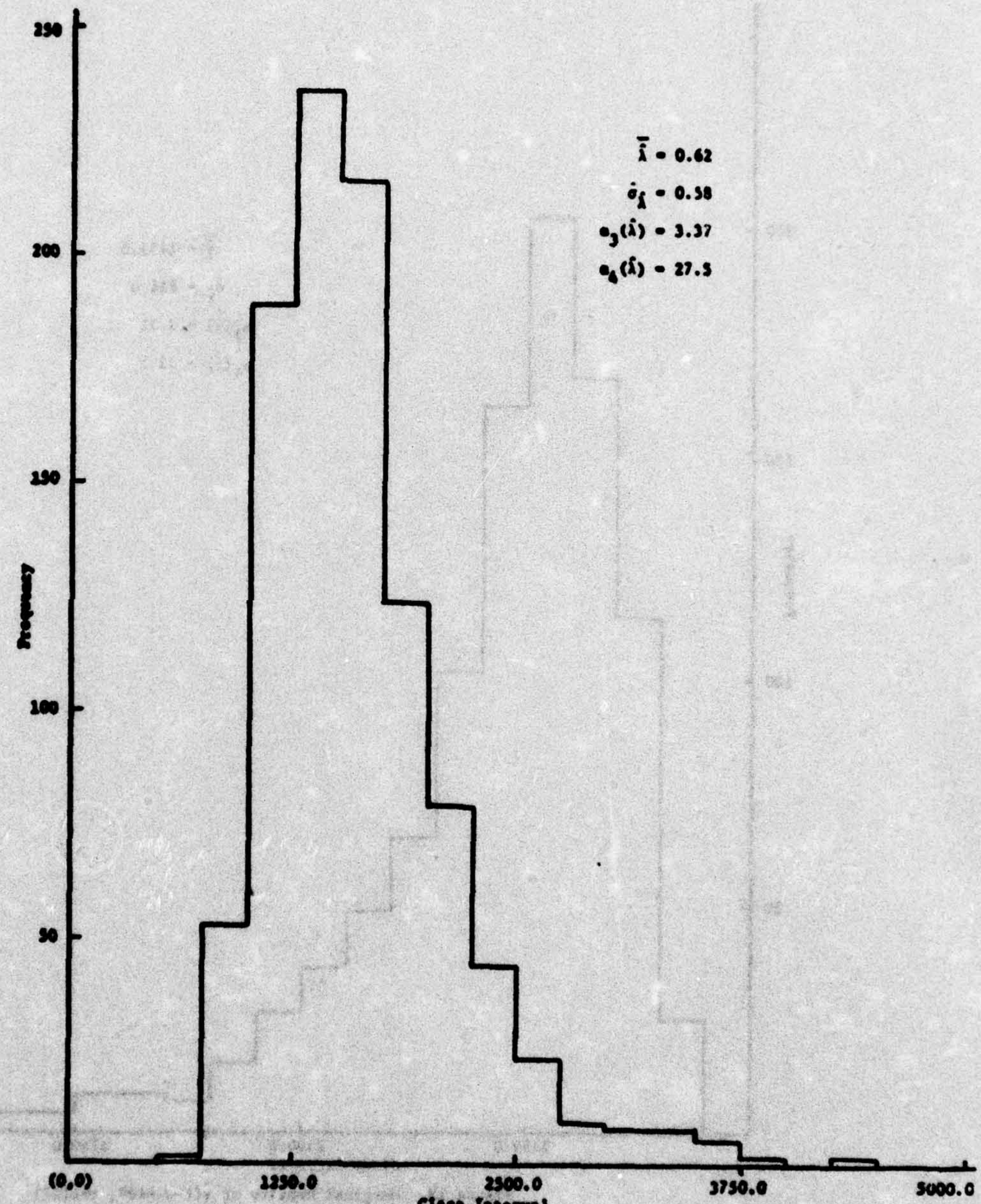


Figure 18. Marginal Density of $\hat{\lambda}$(λ=0.4407, γ=1152)

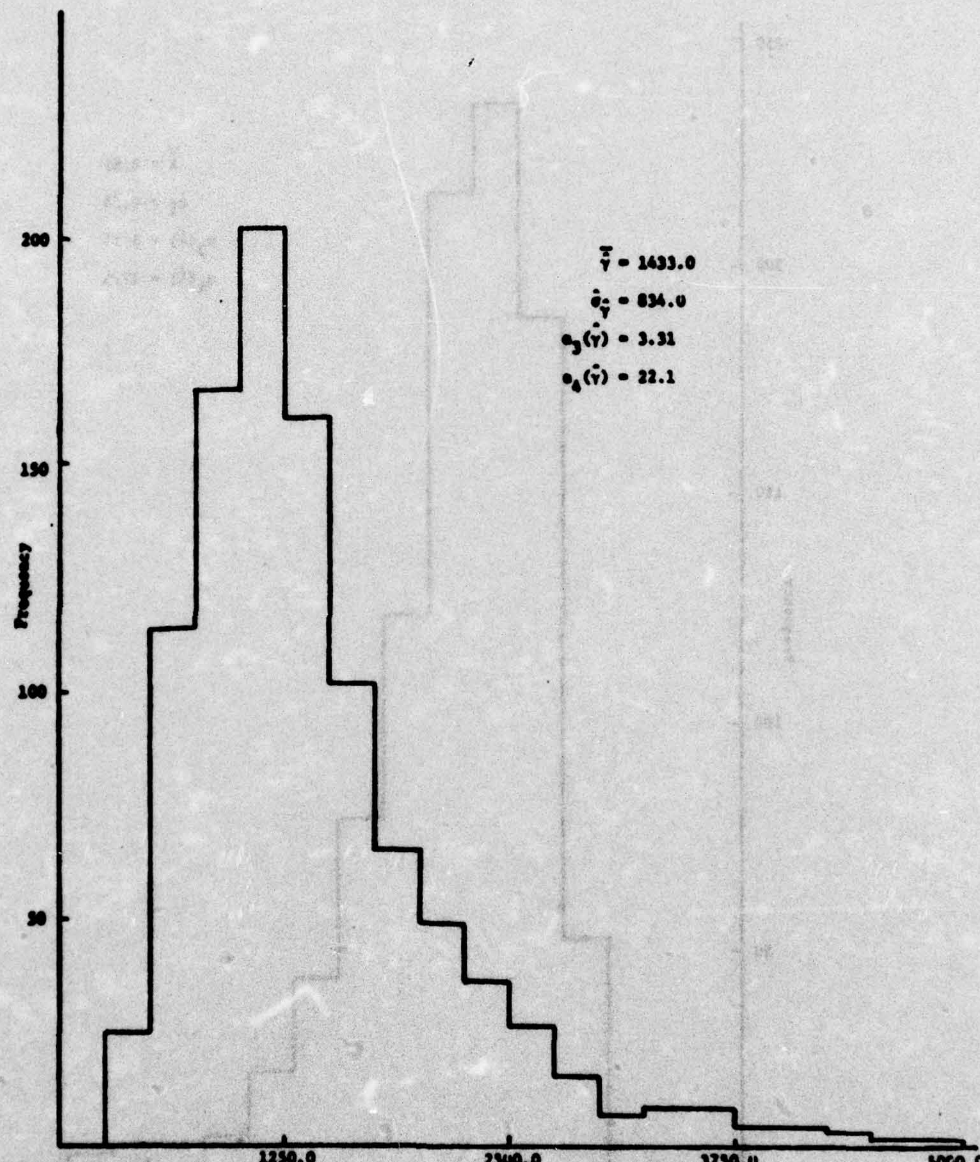


Figure 19 Marginal Density of $\hat{\gamma}(\lambda=0.4407,\ \gamma=1152)$

(ii) The computed averages $\bar{\hat{\lambda}}$ and $\bar{\hat{\gamma}}$ are consistantly larger than λ and γ indicating the small sample bias of maximum likelihood estimates. The bias remains the same for 200 simulations and 1000 simulations and depends upon the number of observations in each simulation and not on the number of simulations.

(iii) The small sample average standard deviations $\hat{\sigma}_{\hat{\lambda}}$ and $\hat{\sigma}_{\hat{\gamma}}$ are larger than the standard deviations obtained from the information matrix.

(iv) The marginal distributions of $\hat{\lambda}$ and $\hat{\gamma}$ are skewed to the right as also indicated by the positive values of skewness.

(v) The marginal distributions are generally leptokurtic. 200 simulations are insufficient to estimate α_4 as evidenced by the fact that $\alpha_4(\hat{\lambda}) = 13.6$ for 200 simulations and $\alpha_4(\hat{\lambda}) = 3.37$ for 1000 simulations, for cases (b) and (d), Figures 14 and 18 respectively. This is expected since estimation of higher moments will require larger simulation.

(vi) 200 simulations appear to be sufficient to estimate the joint and the marginal densities since these densities remain essentially the same for 1000 simulations.

Results of sensitivity analyses are discussed later in Section 3.5.

3.3 Likelihood Contours

The likelihood of λ, γ given the data is:

$$L(\lambda;\gamma|x,n_x) = \prod_{x=1}^{K} \left[\frac{(\lambda+x-1)!}{x!(\lambda-1)!\{1-(\frac{\gamma}{T+\gamma})^{\lambda}\}} (\frac{T}{T+\gamma})^{x} (\frac{\gamma}{T+\gamma})\right]^{n_x} \quad \lambda,\gamma > 0 \qquad (10)$$

Given the observed values of x and n_x (see Table 14), the likelihood function can be evaluated for any λ,γ.

The normalized likelihood function is defined as:

$$L^*(\lambda,\gamma|x,n_x) = \frac{L(\lambda,\gamma|x,n_x)}{L(\hat{\lambda},\hat{\gamma}|x,n_x)} \qquad (11)$$

where $L(\hat{\lambda},\hat{\gamma}|x,n_x)$ is the maximum likelihood and clearly $0 < L^*(\lambda,\gamma|x,n_x) < 1$.

For the three sets of data under consideration, plots of constant normalized likelihood are given in Figures 20, 21 and 22. The plots are restricted to positive parameter values and are elliptical in nature. The maximum likelihood estimates can be easily obtained from these plots and are the same as those given before.

The plots may be interpreted as follows. The 0.05 contour implies that λ,γ values on this contour are 1/20th as likely as the maximum likelihood estimates in the light of the observed data. If a uniform prior is assumed on λ and γ, then the likelihood contours are proportional to the highest posterior density regions.

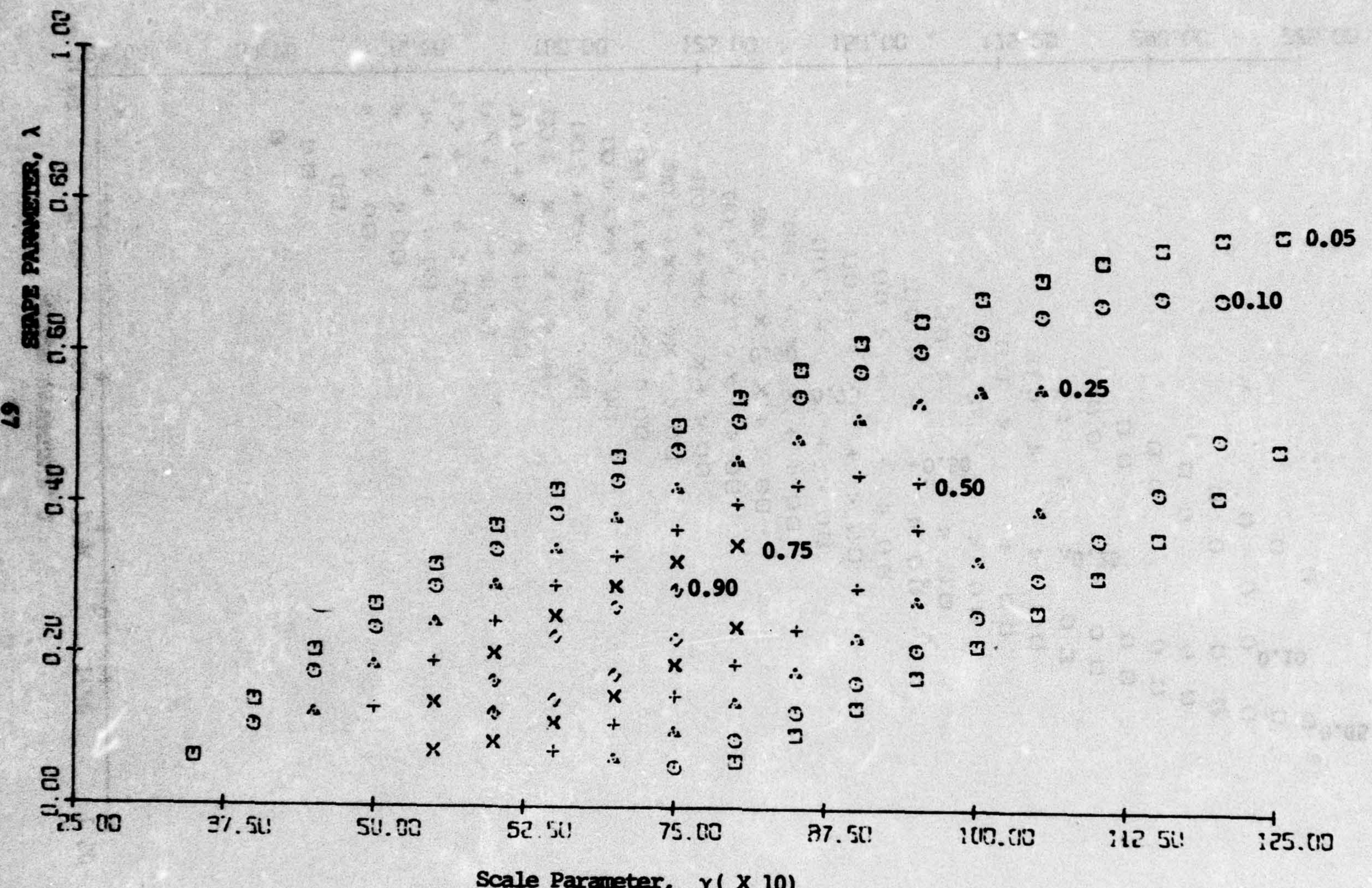


Figure 20. Contours of Normalized Likelihood (Transmitter Data, $\hat{\lambda} = 0.2054$, $\hat{\gamma} = 680$)

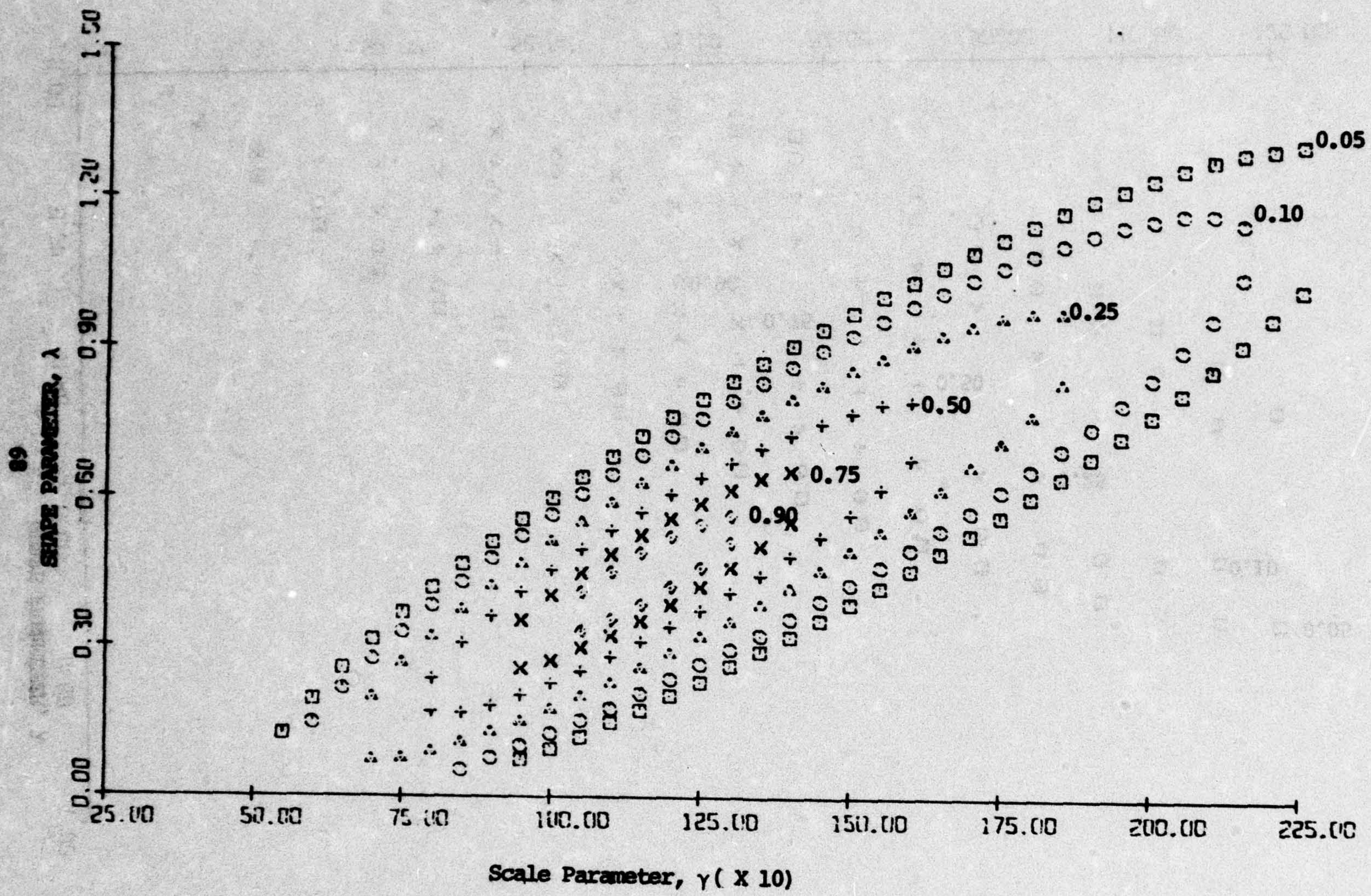


Figure 21. Contours of Normalized Likelihood (Search Indicator Data, $\hat{\lambda} = 0.4407$, $\hat{\gamma} = 1152$)

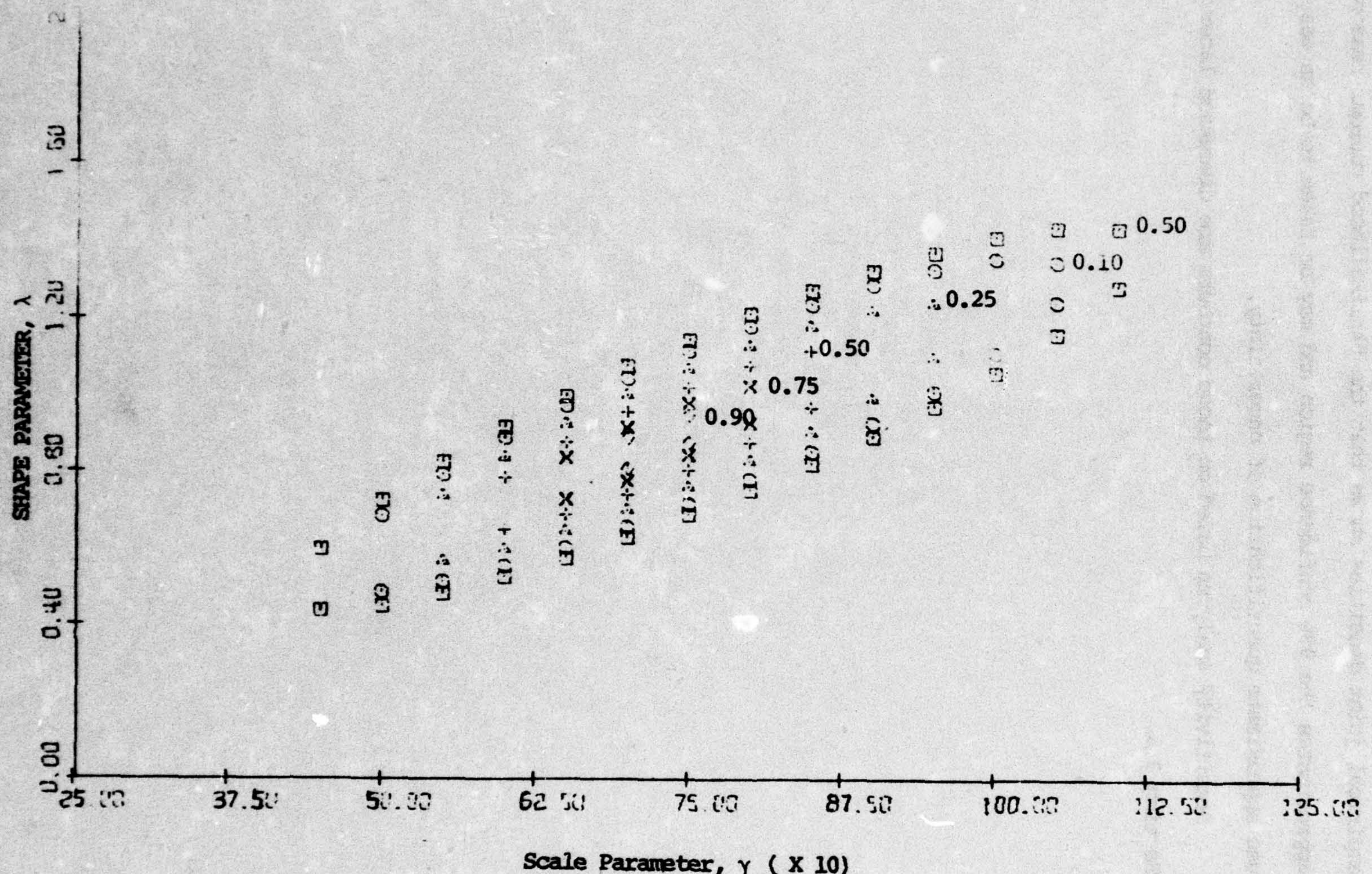


Figure 22. Contours of Normalized Likelihood (Search MVPS Data, $\hat{\lambda} = 0.8646$, $\hat{\gamma} = 714$)

The likelihood contours do not have the confidence region interpretation. However, an empirical comparison of the likelihood contours and the empirical joint densities shows that the 5% likelihood contour reasonably approximates the 95% confidence region and may be taken to be an easy to use approximate quantification of uncertainty.

Sensitivity analyses based on these contours are discussed later in Section 3.5.

3.4 Asymptotic Bivariate Normal Contours

Maximum likelihood estimates are asymptotically normally distributed. Therefore, $\hat{\lambda}$ and $\hat{\gamma}$ have an asymptotic joint bivariate normal distribution given by

$$f(\hat{\lambda},\hat{\gamma})=[2\pi\sqrt{1-\rho^2_{\hat{\lambda},\hat{\gamma}}}]^{-1}\exp[-\frac{1}{2(1-\rho^2_{\hat{\lambda},\hat{\gamma}})}\{(\frac{\hat{\lambda}-\lambda}{\sigma_{\hat{\lambda}}})^2 -2\rho_{\hat{\lambda},\hat{\gamma}}(\frac{\hat{\lambda}-\lambda}{\sigma_{\hat{\lambda}}})(\frac{\hat{\gamma}-\gamma}{\sigma_{\hat{\gamma}}}) + (\frac{\hat{\gamma}-\gamma}{\sigma_{\hat{\gamma}}})^2\}] \tag{12}$$

Contours containing $100(1-\alpha)\%$ of the distribution are given by

$$(\frac{\hat{\lambda}-\lambda}{\sigma_{\hat{\lambda}}})^2 - 2\rho_{\hat{\lambda},\hat{\gamma}}(\frac{\hat{\lambda}-\lambda}{\sigma_{\hat{\lambda}}})(\frac{\hat{\gamma}-\gamma}{\sigma_{\hat{\gamma}}}) + (\frac{\hat{\gamma}-\gamma}{\sigma_{\hat{\gamma}}})^2 = -2(1-\rho^2_{\hat{\lambda},\hat{\gamma}})\log(1-\alpha) \tag{13}$$

Figures 23,24 and 25 show plots of such contours for $\alpha = 0.05$, 010 and 0.1587 for the three cases being considered. The plots are drawn by taking $\sigma_{\hat{\lambda}},\sigma_{\hat{\gamma}}$ and $\rho_{\hat{\lambda},\hat{\gamma}}$ to be numerically equal to $\hat{\sigma}_{\hat{\lambda}},\hat{\sigma}_{\hat{\gamma}}$ and $\hat{\rho}_{\hat{\lambda},\hat{\gamma}}$ respectively.

These contours can be interpreted as asymptotic confidence regions for $\hat{\lambda}$ and $\hat{\gamma}$. A comparison with the small sample results obtained by simulation indicates that these contours may be considered to form a quantification of uncertainty.

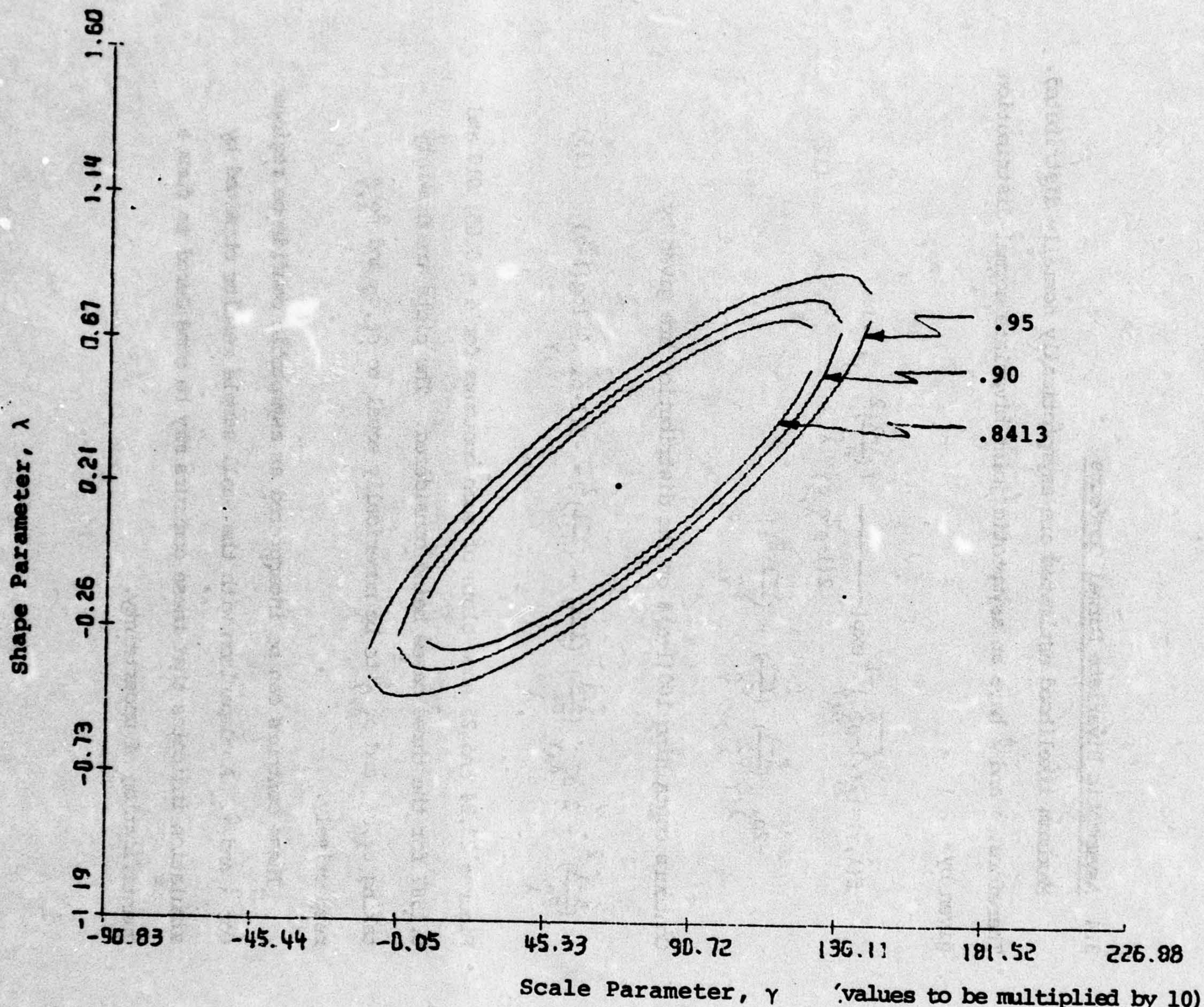


Fig. 23 Contours of Asymptotic Normal Distribution of $(\hat{\lambda}, \hat{\gamma})$ (Transmitter Data)

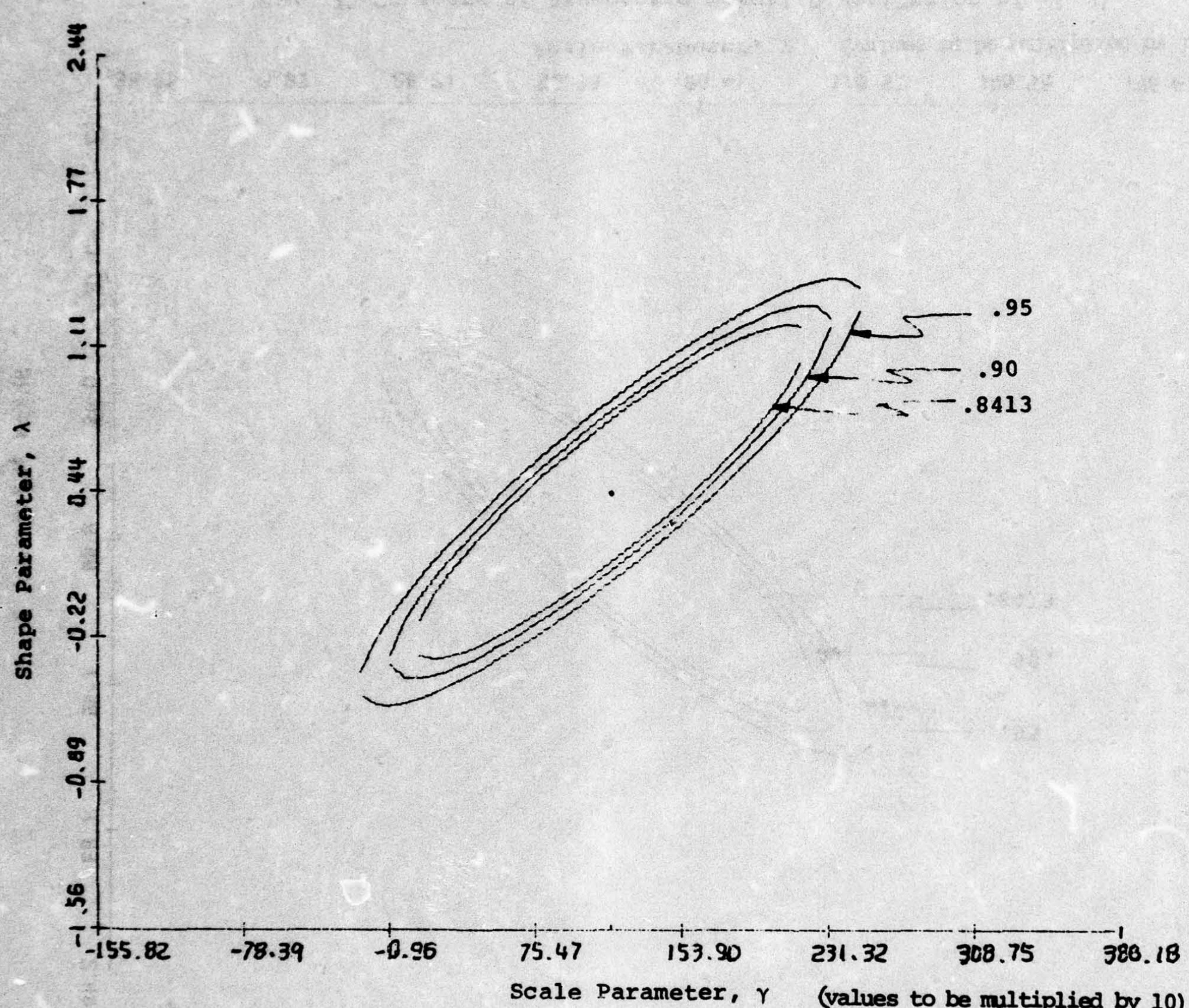


Fig. 24 Contours of Asymptotic Normal Distribution of $(\hat{\lambda}, \hat{\gamma})$
(Search Indicator Data)

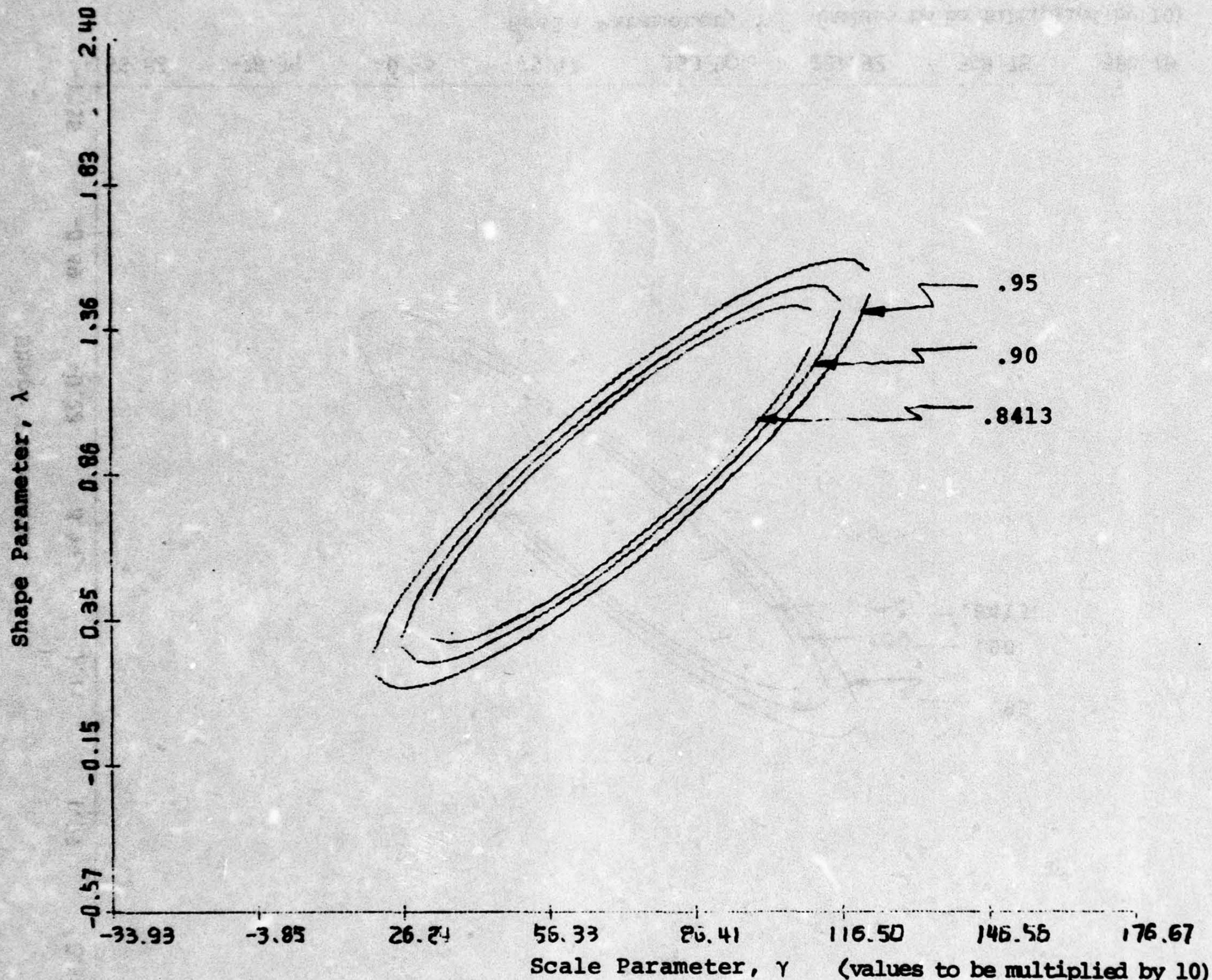


Fig. 25 Contours of Asymptotic Normal Distribution of $(\hat{\lambda},\hat{\gamma})$
(Search MVPS Data)

3.5 Sensitivity Analyses and Discussion

The three approaches discussed above were intended to obtain some quantification of the prior parameters λ and γ. Using the results from these approaches of analysis, the following two types of sensitivity were conducted.

1. For each of the three examples under consideration, λ and γ values along the 95% confidence contour were judiciously selected. For the four risk criteria, namely, (α^*,β^*), $(\bar{\alpha},\bar{\beta})$, $(\bar{\alpha},\beta^*)$ and $(P(R),\beta^*)$, plans were designed to obtain T^* and r^* at the selected parameter values for two discrimination ratios, $K = 2$ and $K = 1.5$. The results are given in Tables 15 to 20.
2. For each of the four risk criteria, plans were designed at $\hat{\lambda}$ and $\hat{\gamma}$. Using the designed T^* and r^* values, the risks were computed at each of the λ,γ values selected above. The results from this analysis are tabulated in Tables 21 to 26.

The first sensitivity analysis gives the effect of the uncertainty associated with the parameters on the designed plans and the second sensitivity analysis gives the effect on risks.

The following observations are made from the results of the above sensitivity analyses as given in Tables 15 to 26.

(i) In general, the designed plans and risks are significantly sensitive to changes in the prior parameters for the data under consideration. Changes in T^* by a factor of 4, changes in r^* by a factor of 2 and changes in risks by a factor of 3 are fairly common.

TABLE 15

SENSITIVITY ANALYSIS FOR T^*, r^* (λ=0.2054, γ=680, θ_0=5600, K=2)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\bar{\alpha}=\bar{\beta}=0.05$		$\bar{\alpha}=\beta^*=0.05$		$P(R)=0.2\beta^*=0.05$	
			r^*	T^*	r^*	T^*	r^*	T^*	r^*	T^*
0.2	450	0.0803	1	0.80–0.89	-	-	1	0.80–1.50	-	-
0.2	1100	0.1964	2	1.00–1.40	1	1.30 1.58	1	0.65–1.55	1	0.25–0.30
0.2054 0.2	680	0.1214	2	1.08–1.63	1	1.18–1.49	1	0.74–1.45	2	1.08–1.15
0.4	600	0.1071	2	1.59–2.07	2	1.60–2.02	2	1.62–2.05	-	-
0.4	1300	0.2321	2	1.42	2	1.76–2.06	2	1.45–2.05	-	-
0.6	750	0.1339	3	3.27–3.79	3	2.64–2.78	4	3.42–3.77	-	-
0.6	1400	0.2500	3	2.24–2.82	3	2.22–2.69	2	2.02–2.38	-	-
0.8	1100	0.1964	2	2.10–2.30	3	2.15–2.40	4	3.08 3.34	-	-
0.8	1400	0.2500	2	2.40–2.60	3	2.20 2.44	4	2.75 3.14	-	-

TABLE 16

SENSITIVITY ANALYSIS FOR T^*, r^* (λ=0.4404, γ=1152, θ_0=6500, K=2)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\bar{\alpha}=\bar{\beta}=0.05$		$\bar{\alpha}=\beta^*=0.05$		$\beta^*=0.05$ P(R)=0.2	
			r^*	T^*	r^*	T^*	r^*	T^*	r^*	T^*
0.2	570	0.0876	1	0.80-0.89	-	-	1	0.80-1.50	-	-
0.2	1180	0.1815	2	1.01-1.42	1	1.26-1.52	1	0.69-1.52	1	0.29-0.34
0.4	750	0.1153	2	1.59-2.07	2	1.63-2.04	2	1.60-2.03	-	-
0.4	1500	0.2307	2	1.42	2	1.76-2.06	2	1.49-2.04	-	-
0.440 0.4	1152	0.1772	2	1.50-1.66	2	1.72-2.05	2	1.52-2.00	-	-
0.6	950	0.1461	2	2.68-3.06	3	2.54-2.66	3	2.66-3.02	-	-
0.6	1850	0.2846	3	2.16-2.58	3	2.27-2.73	2	1.70-1.77	-	-
0.8	1250	0.1923	2	2.13-2.38	3	2.15-2.40	4	3.08-3.34	-	-
0.8	2150	0.3307	3	2.14-2.35	3	2.29-2.48	3	2.58-2.80	-	-
1.0	1600	0.2461	2	2.52-3.02	3	2.09-2.21	5	3.84-4.02	-	-
1.0	2350	0.3615	3	2.48-3.04	3	2.27-2.39	4	3.06-3.20	-	-
1.5	2500	0.3846	2	2.70-3.55	4	2.64-2.89	3	1.98-2.06	-	-

TABLE 17

SENSITIVITY ANALYSIS FOR T^*, r^* (λ=0.8646, Υ=714, θ_0=1000, K=2)

λ	Υ	$\Upsilon^* = \frac{\Upsilon}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\alpha=\beta=0.05$		$\alpha=\beta^*=0.05$		$\beta^*=0.05$, R(R)=.2	
			r^*	T^*	r^*	T^*	r^*	T^*	r^*	T^*
0.6	520	0.520	3	1.83-1.97	3	2.34-2.82	2	1.40-1.82	5	2.63-2.67
0.6	700	0.700	3	1.45-1.51	2	1.84-1.88	1	0.74-0.88	1	0.74-0.98
0.8	600	0.600	4	2.44-2.88	3	2.39-2.57	2	2.55-2.64	-	-
0.8	850	0.850	4	2.04-2.48	3	2.36-2.66	2	1.24-1.70	2	1.24-1.46
0.8646 0.8	714	0.714	4	2.20-2.24	3	2.38-2.60	2	1.85-2.15	-	-
1.0	650	0.650	4	2.80-3.16	3	2.44-2.48	3	2.20-2.45	-	-
1.0	1000	1.000	4	2.02-2.31	2	1.72	2	1.16-1.61	1	0.48-0.82
1.5	800	0.800	3	2.71-2.73	4	2.88-3.00	5	3.70-3.92	-	-
1.5	1200	1.200	4	2.75-2.78	4	2.90-3.10	4	2.74-3.10	-	-

TABLE 18

SENSITIVITY ANALYSIS FOR T^*, r^* ($\hat{\lambda}=0.2054$, $\hat{\gamma}=680$, $\theta_0=5600$, $K=1.5$)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\bar{\alpha}=\bar{\beta}=0.05$		$\bar{\alpha}=\beta^*=0.05$		$\alpha^*=0.05, P(E)=0.2$	
			r^*	T^*	r^*	T^*	r^*	T^*	r^*	T^*
0.2	450	0.0803	2	1.60–1.97	1	1.52	1	1.11–1.50	-	-
0.2	1100	0.1964	3	1.85–2.30	2	2.38 2.82	1	0.95–1.52	1	0.96–1.11
0.2054 0.2	680	0.1214	2	1.53–1.70	2	2.28–2.81	1	1.05–1.53	-	-
0.4	600	0.1071	2	2.25–2.56	2	2.00–2.16	3	2.84–3.25	-	-
0.4	1300	0.2321	4	3.12–3.44	3	3.04–3.12	3	2.58–3.08	-	-
0.6	750	0.1339	3	3.28–3.79	3	2.66–2.78	5	4.52–4.83	-	-
0.6	1400	0.2500	5	4.40–4.90	4	3.72	5	4.42–4.78	-	-
0.8	1100	0.1964	-	-	-	-	-	-	-	-
0.8	1400	0.2500	4	4.12–4.26	5	4.18–4.39	6	5.36 5.42	-	-

TABLE 19

SENSITIVITY ANALYSIS FOR T*, r* (λ=0.4407, γ=1152, θ_0=7000, K=1.5)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\bar{\alpha}=\bar{\beta}=0.05$		$\bar{\alpha}=\beta^*=0.05$		$\beta^*=0.05$, P(R)=0.2	
			r*	T*	r*	T*	r*	T*	r*	T*
0.2	570	0.0814	2	1.60–1.97	1	1.52	1	1.11–1.50	-	-
0.2	1180	0.1685	3	1.91–2.42	2	2.35–2.79	1	1.00–1.52	2	1.42–1.56
0.4	750	0.1071	2	2.25–2.56	2	2.00–2.16	3	2.84–3.25	-	-
0.4	1500	0.2142	4	3.12–3.44	3	2.96–3.10	3	2.62–3.08	-	-
0.4407 0.4	1152	0.1645	3	2.72–2.82	3	2.88–3.07	3	2.72–3.08	-	-
0.6	950	0.1357	3	3.28–3.79	3	2.66–2.78	5	4.52–4.83	-	-
0.6	1850	0.2642	5	4.40–4.90	4	3.72	5	4.42–4.78	-	-
0.8	1250	0.1785	3	3.48 3.86	4	3.69–4.27	6	5.78–5.95	-	-
0.8	2150	0.3071	4	4.16–4.35	5	4.22–4.48	6	5.32–5.40	-	-
1.0	1600	0.2285	4	4.21–4.46	5	3.84–3.96	8	7.66–7.72	-	-
1.0	2350	0.3357	4	4.68–5.12	5	4.36 4.45	8	6.48–6.61	-	-
1.5	2500	0.3471	-	-	-	-	-	-	-	-

TABLE 20

SENSITIVITY ANALYSIS FOR T*, r* ($\hat{\lambda}$=0.8646, $\hat{\gamma}$=714, θ_0 =1000, K=1.5)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	$\alpha^*=\beta^*=0.05$		$\bar{\alpha}=\bar{\beta}=0.05$		$\bar{\alpha}=\beta^*=0.05$		$\beta^*=0.05$ P(R)=.2	
			r*	T*	r*	T*	r*	T*	r*	T*
0.6	520	0.520	6	4.37-4.74	5	4.68-4.95	3	2.66-2.82	-	-
0.6	700	0.700	6	4.08-4.48	5	4.72-5.08	3	2.44-2.90	5	3.55
0.8	600	0.600	6	4.87-5.11	6	5.24 5.56	5	4.28-4.53	-	-
0.8	850	0.850	7	4.94-5.28	6	5.41-5.80	4	3.21-3.66	-	-
0.8648 0.8	714	0.714	7	5.10-5.40	6	5.35-5.87	4	3.40-3.75	-	-
1.0	650	0.650	6	5.32-5.54	6	5.17-5.26	7	5.98-6.27	-	-
1.0	1000	1.000	7	4.92	6	5.50-5.57	4	3.20-3.52	7	4.91-4.99
1.5	800	0.800	8	6.62-7.23	8	6.67-6.84	7	3.64-4.83	-	-
1.5	1200	1.200	9	6.65-6.89	8	6.81-7.02	6	4.88-5.09	-	-

TABLE 21

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.2054, $\hat{\gamma}$=680, θ_0=5600, K=2)

λ	γ	$\gamma^* = \frac{\gamma}{\sigma_0}$	T^*=1.08, r^*=2		T^*=1.18, r^*=1		T^*=0.74, r^*=1	
			α^*	β^*	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	β^*
0.2	450	0.0803	0.011	0.057	0.034	0.045	0.015	0.057
0.2	1100	0.1964	0.037	0.042	0.035	0.058	0.016	0.041
0.2054 0.2	680	0.1214	0.022	0.053	0.036	0.051	0.017	0.052
0.4	600	0.1071	0.009	0.113	0.062	0.046	0.027	0.114
0.4	1300	0.2321	0.030	0.085	0.063	0.059	0.030	0.084
0.6	750	0.1339	0.009	0.168	0.086	0.046	0.038	0.172
0.6	1400	0.2500	0.023	0.132	0.087	0.059	0.042	0.133
0.8	1100	0.1964	0.013	0.207	0.111	0.051	0.054	0.214
0.8	1400	0.2500	0.017	0.188	0.139	0.057	0.052	0.192

TABLE 22

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.4409, $\hat{\gamma}$=1152, θ_0=6500, K=2)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	T^*=.150, r^*=2		T^*=1.72, r^*=2		T^*=1.52, r^*=2	
			α^*	β^*	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	β^*
0.2	570	0.0876	0.031	0.029	0.018	0.042	0.013	0.028
0.2	1180	0.1815	0.058	0.023	0.019	0.052	0.015	0.022
0.4	750	0.1153	0.025	0.059	0.034	0.042	0.025	0.058
0.4	1500	0.2307	0.051	0.046	0.036	0.054	0.027	0.045
0.4407 0.4	1152	0.1772	0.038	0.058	0.040	0.049	0.030	0.057
0.6	950	0.1461	0.022	0.091	0.050	0.042	0.038	0.089
0.6	1850	0.2846	0.045	0.069	0.049	0.056	0.038	0.067
0.8	1250	0.1923	0.022	0.121	0.066	0.045	0.051	0.118
0.8	2150	0.3307	0.039	0.094	0.062	0.057	0.047	0.091
1.0	1600	0.2461	0.020	0.149	0.078	0.047	0.060	0.145
1.0	2350	0.3615	0.031	0.122	0.073	0.057	0.056	0.119
1.5	2500	0.3846	0.015	0.213			0.074	0.208

TABLE 23

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.8646, $\hat{\gamma}$=714, θ_0=1000, K=2)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	T^*=2.2, r^*=3		T^*=2.38, r^*=3		T^*=1.85, r^*=2	
			α^*	β^*	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	β^*
0.6	520	0.520	0.027	0.051	0.032	0.048	0.052	0.026
0.6	700	0.700	0.041	0.036	0.030	0.047	0.049	0.018
0.8	600	0.600	0.025	0.065	0.041	0.049	0.066	0.035
0.8	850	0.850	0.043	0.041	0.037	0.047	0.060	0.022
0.8646 0.8	714	0.714	0.030	0.060	0.042	0.050	0.068	0.032
1.0	650	0.650	0.022	0.083	0.049	0.051	0.078	0.046
1.0	1000	7.000	0.041	0.046	0.044	0.047	0.070	0.024
1.5	800	0.800	0.017	0.126	0.067	0.054	0.103	0.075
1.5	1200	1.200	0.038	0.069	0.060	0.050	0.094	0.039

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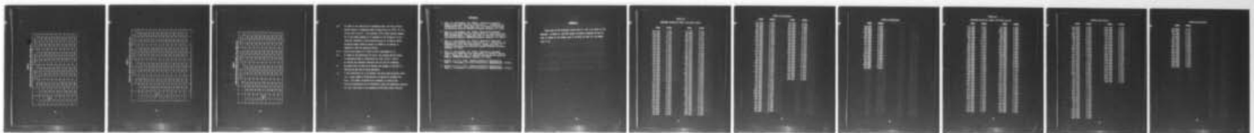

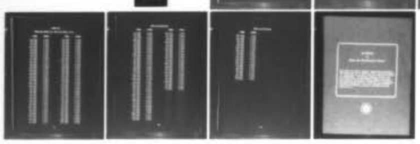

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TABLE 24

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.2054, $\hat{\gamma}$=680, θ_0=5600, K=1.5)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	T^*=1.53, r^*=2		T^*=2.28, r^*=2		T^*=1.05, r^*=1	
			α^*	β^*	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	β^*
0.2	450	0.0803	0.029	0.054	0.033	0.041	0.028	0.054
0.2	1100	0.1964	0.064	0.043	0.034	0.058	0.029	0.043
0.2054 0.2	680	0.1214	0.043	0.051	0.036	0.048	0.030	0.051
0.4	600	0.1071	0.024	0.109	0.062	0.040	0.051	0.112
0.4	1300	0.2321	0.053	0.088	0.063	0.057	0.053	0.089
0.6	750	0.1339	0.020	0.164	0.087	0.038	0.071	0.171
0.6	1400	0.2500	0.041	0.137	0.088	0.054	0.073	0.141
0.8	1100	0.1964	0.023	0.209	0.114	0.042	0.093	0.218
0.8	1400	0.2500	0.030	0.194	0.112	0.049	0.091	0.202

TABLE 25

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.4407, $\hat{\gamma}$=1152, θ_0=7000, K=1.5)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	T^*=2.72, r^*=3		T^*=2.88, r^*=3		T^*=2.72, r^*=3	
			$\alpha*$	$\beta*$	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\beta*$
0.2	570	0.0814	0.037	0.026	0.021	0.042	0.018	0.026
0.2	1180	0.1685	0.064	0.022	0.022	0.056	0.019	0.022
0.4	750	0.1071	0.030	0.054	0.039	0.041	0.033	0.054
0.4	1500	0.2142	0.056	0.046	0.042	0.057	0.037	0.046
0.4407 0.4	1152	0.1645	0.043	0.055	0.047	0.049	0.041	0.055
0.6	950	0.1357	0.025	0.084	0.057	0.039	0.049	0.084
0.6	1850	0.2642	0.050	0.070	0.059	0.057	0.051	0.070
0.8	1250	0.1785	0.025	0.114	0.078	0.040	0.068	0.114
0.8	2150	0.3070	0.043	0.096	0.074	0.056	0.064	0.096
1.0	1600	0.2285	0.023	0.142	0.094	0.041	0.082	0.142
1.0	2350	0.3357	0.035	0.125	0.088	0.054	0.076	0.125
1.5	2500	0.3571	0.017	0.214	0.119	0.043	0.104	0.214

TABLE 26

SENSITIVITY ANALYSIS FOR RISKS ($\hat{\lambda}$=0.8646, $\hat{\gamma}$=714, θ_0=1000, K=1.5)

λ	γ	$\gamma^* = \frac{\gamma}{\theta_0}$	T^*=5.10, r^*=7		T^*=5.35, r^*=6		T^*=3.40, r^*=4	
			α^*	β^*	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	β^*
0.6	520	0.52	0.031	0.046	0.033	0.046	0.035	0.044
0.6	700	0.70	0.044	0.036	0.031	0.051	0.032	0.034
0.8	600	0.60	0.030	0.061	0.043	0.047	0.044	0.059
0.8	850	0.85	0.046	0.044	0.038	0.053	0.040	0.042
0.8626 0.8	714	0.714	0.034	0.059	0.044	0.050	0.045	0.057
7.0	650	0.65	0.026	0.078	0.052	0.047	0.053	0.077
7.0	1000	1.00	0.048	0.051	0.045	0.055	0.047	0.049
1.5	800	0.80	0.020	0.121	0.072	0.047	0.072	0.122
1.5	1200	1.20	0.041	0.080	0.064	0.056	0.065	0.078

(ii) In terms of the sensitivity of designed plans, the risk criteria may be ranked in increasing order of sensitivity as $(\bar{\alpha},\bar{\beta})$, $(\bar{\alpha}, \beta^*)$, (α^*,β^*) and $(P(R),\beta^*)$. The criterion $(\bar{\alpha},\beta^*)$ shows smaller changes in T^* and larger changes in r^* compared to the criterion (α^*,β^*). $(P(R),\beta^*)$ criterion is very sensitive since it is easy to obtain situations where infinite testing is needed or no testing is required to meet the desired criteria.

(iii) Part of the change in T^* above is due to the change in r^*.

(iv) In terms of the sensitivity of risks, the criteria may be ranked in increasing order or sensitivity as $(\bar{\alpha},\bar{\beta})$, $(\bar{\alpha},\beta^*)$, (α^*,β^*). No results are currently available for the $P(R),\beta^*$ criterion.

(v) The sensitivity is such as to balance the changes in the cost of testing and the cost of wrong decisions.

(vi) If the sensitivity is to be reduced, the prior must be better known, i.e., larger number of observations is required to estimate the prior. The number of observations necessary to yield a prespecified sensitivity can be obtained by using the simulation approach, the likelihood plots or the asymptotic bivariate normal contours.

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APPENDIX A

Three sets of 200 estimated values each of λ and γ are given in this Appendix. In Table A.1 the 200 values are based on samples of size 50 each, in Table A.2 the sample size is 55 while in Table A.3 the sample size is 74.

TABLE A.1

SIMULATED VALUES OF $\hat{\lambda}$ AND $\hat{\gamma}$ (λ=0.2054, γ=680)

GAMMA	LAMBDA	GAMMA	LAMBDA
1626.2998	1.2906	470.2000	-0.0349
1163.7000	0.6500	454.8999	0.1662
1188.8999	0.5156	800.3999	0.2258
338.8999	-0.0752	887.0000	0.4657
1053.3999	0.4376	621.7998	0.1914
644.5999	0.1307	1199.3999	0.5029
488.2000	-0.0717	671.3999	0.1741
1404.5999	0.5922	261.3999	-0.2296
819.0000	0.2997	643.7998	0.1623
885.2000	0.4050	507.7998	0.0958
1298.2998	0.6401	587.7000	0.1637
412.7998	-0.1883	594.0000	0.1707
978.3999	0.7273	1430.0000	0.5358
1383.0000	0.6035	386.2998	-0.0608
730.5000	0.4183	1308.7998	0.9830
906.8999	0.1529	1094.8999	0.2526
691.5000	0.2390	394.2000	-0.1483
555.0000	0.2494	817.2000	0.2079
1254.5999	0.6296	1160.3999	0.3279
785.2998	0.1398	810.7000	0.1061
723.2000	0.2949	573.5000	0.0935
748.2000	0.2448	2302.5000	1.5078
604.2000	0.1214	374.7998	-0.1243
655.5000	0.2488	1044.8999	0.9630
793.0999	0.2512	1107.5999	0.5892
769.3999	-0.0403	905.3999	0.0970
734.0000	0.1953	566.2998	0.2369
383.0000	-0.1965	1267.0000	0.3970
713.7998	0.0072	451.5000	0.0656
1849.5000	1.5158	687.2000	0.2565
916.5000	0.6314	522.8999	0.0355
809.5999	0.3364	853.7000	0.4314
660.3999	0.0665	1798.3999	0.6599
763.0000	0.3659	589.7998	0.1855
2387.8999	1.6382	499.5999	0.1583
532.7000	-0.0325	2011.3999	0.8416
762.0000	0.4822	326.5999	0.0645
798.7998	0.3744	719.0999	0.4675
936.0000	0.5954	259.0999	-0.3032
5323.8984	2.6631	447.7000	0.0369
754.8999	0.1902	1302.2998	0.7074
1117.0999	0.5014	705.0999	0.3410
684.0999	0.6191	754.2998	-0.0156
719.0000	0.1013	819.2998	0.3208
545.0999	0.1531	700.7998	0.2490
415.5000	0.0281	1563.0000	1.0285
480.8999	-0.1029	487.2000	-0.0259

TABLE A.1(Continued)

GAMMA	LAMBDA	GAMMA	LAMBDA
1556.7000	0.8256	692.0000	0.0150
1962.2998	1.0254	529.7998	0.1260
809.5000	0.2210	592.7998	0.0888
640.0000	0.2772	646.8999	0.2082
611.0999	0.0703	1071.5000	1.2839
1084.5000	0.9083	821.8999	0.2456
1111.2000	0.4549	1266.7998	0.5211
992.2000	0.3611	567.0000	0.0370
1031.5000	0.4930	1153.7998	0.4305
510.7000	0.0270	694.5999	-0.0950
3705.2000	1.9025	395.0999	0.1135
474.7000	0.0098	962.5999	0.2435
1242.5000	0.7404	860.0999	0.4695
238.6000	-0.2872	816.2000	0.0748
971.7000	0.3659	685.0999	0.3027
970.7000	0.2967	569.0999	-0.0184
1855.0000	0.9946	1208.3999	0.4195
331.8999	-0.2345	423.7998	-0.0040
827.0999	0.5062	360.2000	-0.0028
697.0000	0.1770	557.2998	0.0563
765.0999	0.2246	514.5000	-0.0260
385.0999	-0.1986	607.2998	0.2537
1305.5000	0.2152	753.5999	0.1575
743.7000	0.3842	494.3999	0.0407
1376.5999	0.4443	615.0999	-0.0560
582.5999	0.4101	658.2000	0.3234
464.0000	0.0483	768.0999	0.1770
2172.5000	1.0529	475.7000	0.0915
920.5999	0.3134	516.5000	0.1115
472.7998	0.0673	802.7000	0.3845
555.0999	0.1737	765.8999	0.1296
346.3999	-0.2799	1188.2998	0.4222
848.7000	0.2901		
433.0000	0.2708		
534.7000	-0.0929		
452.0000	-0.0588		
436.0999	0.0148		
1201.7998	0.6266		
1007.8999	0.5750		
2455.5999	1.5037		
302.7000	-0.0306		
1207.2000	0.6057		
608.5999	0.2744		
717.2000	0.2996		
843.5000	0.4082		
401.7998	-0.0194		
595.7998	0.2308		
838.7998	0.1926		
943.7000	0.3042		
810.5000	0.2416		

TABLE A.1(Continued)

GAMMA	LAMBDA
948.2998	0.0787
808.8999	0.4974
887.8999	0.3538
2238.0999	1.7466
608.7998	0.2328
729.7998	0.4858
1050.2998	0.5279
534.0999	0.2690
553.3999	0.1112
527.8999	0.1504
961.7000	0.3197
771.5999	0.1280
915.2998	0.1516
439.7998	-0.1480
505.0999	0.0980
950.0000	0.5027
1414.5000	0.3388
530.8999	0.1046
415.7000	0.0260
989.5999	0.3435
438.3999	0.1724
1298.0999	0.5174
327.0999	-0.1732
882.8999	0.6357

TABLE A.2

SIMULATED VALUES OF $\hat{\lambda}$ AND $\hat{\gamma}$ (λ=0.4407, γ=1152)

GAMMA	LAMBDA	GAMMA	LAMBDA
1702.2000	0.6841	737.0000	0.1345
467.7000	0.0101	1567.2000	0.3940
780.2998	0.4946	1081.5999	0.3940
2371.3999	1.1299	1379.0000	0.6238
1710.2998	1.0923	639.7998	0.2651
1303.7998	0.8202	789.2998	0.4063
900.5000	0.2266	886.2000	0.4401
561.5999	0.0343	1517.8999	0.8203
435.3999	-0.2675	1287.3999	0.4815
431.3999	-0.1738	650.7000	0.1680
1016.0000	0.3795	1457.2998	0.3480
895.2998	0.0839	1937.7000	1.0283
1004.0000	0.2978	1426.0999	0.3494
1599.3999	0.5365	1539.0999	0.4427
1434.7000	0.5740	3451.8999	2.0869
918.0999	0.3855	1398.0999	0.6663
1166.5999	0.4846	828.0000	0.2882
820.2000	0.2692	1375.5999	0.8634
1423.0000	0.4880	1449.7000	0.8176
3209.8999	1.6547	845.5999	0.2560
1114.8999	0.5752	409.7998	-0.1160
1358.0999	0.7036	1012.0000	0.0728
1256.0000	0.5999	647.5000	-0.0392
1129.7998	0.5222	873.5000	0.1372
2508.0000	1.2068	948.5999	0.4588
828.8999	0.3419	1051.7998	0.3737
1580.0999	0.8331	1127.5999	0.4132
1136.5000	0.4125	3291.2998	2.0623
1140.5999	0.2942	1310.2000	0.6711
836.7000	0.1015	874.2998	0.4280
1004.7998	0.3558	5752.0977	3.2755
1917.0000	0.4030	1151.2998	0.3762
1093.2998	0.2351	1648.0999	1.2471
1455.0000	0.4605	1501.2998	0.6825
941.5000	0.2675	1001.2998	0.1454
2580.3999	1.2396	1288.2998	0.3462
1475.5999	0.4539	1431.0000	0.6838
2319.5000	1.4120	1207.5000	0.3858
604.0999	-0.0280	913.2998	0.1754
1096.5999	0.5077	1567.3999	0.6419
2516.0000	1.4802	1842.5000	1.1328
650.5000	0.0346	550.5000	-0.0712
1107.8999	0.2620	1424.8999	0.6789
1710.5999	0.8435	1064.8999	0.1659
1556.2000	0.7049	2475.5000	1.2642
2113.5000	1.0854	1035.7998	0.5156
1139.3999	0.3831	2233.0000	0.8375
4418.0000	2.2437	569.5999	0.0236

TABLE A.2 (Continued)

GAMMA	LAMBDA	GAMMA	LAMBDA
1071.5000	0.3751	2333.7000	1.2734
1586.8999	0.5064	1048.7000	0.4293
793.7000	0.1489	1069.7000	0.3281
1822.5000	0.7558	610.0999	0.0235
1854.5999	0.9419	1109.7000	0.2634
893.0000	0.2793	944.7998	0.3248
1399.8999	0.8775	759.8999	-0.0230
605.2000	-0.0116	1171.0999	0.7804
521.5000	0.0624	794.0000	0.3408
789.7000	0.2805	1021.2000	0.4678
2193.0999	0.7700	847.5000	0.1100
1214.3999	0.4833	2632.5000	1.5680
1852.2998	0.8940	1517.8999	0.7728
2335.2000	0.9891	977.0000	0.4338
475.0000	0.0224	1844.2000	0.4054
864.5000	0.1368	1084.3999	0.5113
1642.7000	0.8328	8633.7969	4.9344
1313.2998	0.7155	828.5000	0.1598
957.2998	0.2597	1599.7000	0.7686
2903.5999	1.5218	1318.3999	0.5314
1301.2998	0.5616	2176.0000	0.5675
1035.0999	0.4596	1131.3999	0.5977
1867.0000	0.6135	1485.8999	0.6317
2777.2998	1.2009	2743.8999	1.7140
1176.7998	0.5933	1637.5000	0.8287
2419.0999	0.8406	1275.0000	0.3644
1899.8999	0.9993	1339.5999	0.4568
770.7998	0.2453	2236.7998	1.5002
911.2998	0.3599	5005.7969	2.0796
1120.5999	0.6521	1140.0999	0.5463
565.5999	0.0567	966.7000	0.3438
1953.0000	0.9921		
999.7000	0.5474		
1597.3999	0.8965		
1766.5000	1.1106		
1996.5999	0.7723		
1847.7998	1.3600		
1766.3999	1.1518		
2491.5000	1.6316		
1039.0000	0.5322		
804.8999	0.4500		
909.0999	0.0794		
643.2000	0.0280		
642.2998	0.0902		
874.2000	0.2370		
1401.3999	0.5301		
1192.0000	0.5453		
1719.8999	1.0397		
801.0999	0.3201		
1769.2998	1.0396		
3615.7998	2.6707		

TABLE A.2(Continued)

GAMMA	LAMBDA
1105.8999	0.3723
1685.8999	0.7843
857.2000	-0.1207
939.7000	0.2314
724.2000	0.0179
883.0000	0.5012
1352.2998	0.4283
1245.3999	0.5251
536.3999	-0.1083
1256.2998	0.5000
1597.7998	0.8075
841.2998	0.0163
1090.5000	0.3906
1083.5000	0.5674
2995.2998	1.7051
1633.7998	0.9467
1458.5000	0.7615
3508.2998	2.3385
1290.8999	0.7603
1283.0999	0.6805
1285.2998	0.6579
1011.5999	0.3402

TABLE A.3

SIMULATED VALUES OF $\hat{\lambda}$ AND $\hat{\gamma}$ (λ=0.8626, γ=714)

GAMMA	LAMBDA	GAMMA	LAMBDA
1129.7998	1.2922	664.8999	0.7818
698.5000	0.6776	1241.8999	1.3697
691.7000	0.6410	709.2998	1.0558
794.0000	1.0091	801.5000	0.6861
1044.0999	1.3207	667.2998	0.6105
872.0000	1.0300	666.2998	0.7072
913.7998	1.2115	905.0999	1.0029
999.0999	1.2795	668.0000	0.5642
565.5000	0.7024	861.2998	1.2500
795.2998	1.0477	970.7000	1.2543
767.5000	0.9830	449.0000	0.3235
792.0000	1.0922	1087.0999	1.0042
1237.5999	1.5421	1543.5999	1.8180
818.7998	1.0479	912.5000	1.2924
325.0000	0.1347	723.0000	0.9620
885.0000	0.8389	806.0000	0.8293
1129.2998	1.3587	805.7998	0.9857
1652.2000	1.5372	1177.2998	1.1297
635.8999	0.5439	600.2998	0.6387
1048.0000	1.3599	725.2000	0.7326
954.2000	1.0195	2072.7000	2.3643
1249.2998	1.6217	977.0999	1.0214
699.2000	0.9969	699.5000	0.7620
669.7998	0.8840	859.0999	1.0550
1037.7000	1.1893	537.7000	0.4938
1007.3999	1.3464	910.2000	1.4206
1148.3999	1.6338	1707.8999	1.8741
963.7998	1.1645	813.2998	0.9602
722.2000	1.0797	821.7998	1.0376
809.0000	0.9003	687.0999	0.8009
581.3999	0.6610	705.8999	0.9951
792.5999	1.3197	783.5999	1.0594
699.7000	0.7048	697.7998	0.9747
806.3999	1.2531	1137.2000	1.4718
541.8999	0.6337	709.2000	0.7618
884.3999	1.4058	694.3999	0.7746
825.0999	0.6945	922.3999	1.4652
1507.5000	1.6750	945.5000	1.0305
1047.7000	1.0557	1157.7000	1.3907
858.7998	0.9286	805.2000	0.8406
864.5999	1.5309	897.8999	1.1198
790.2998	0.9513	1298.5999	1.6937
1050.7998	0.8512	973.7000	1.2667
772.2998	0.9723	963.2000	1.0230
1307.7000	1.6148	638.7000	0.8663
747.5999	0.7406	1046.5000	1.2430
544.5000	0.4425	702.0999	0.8391

TABLE A.3(Continued)

GAMMA	LAMBDA	GAMMA	LAMBDA
671.0999	1.0229	1131.7000	1.2632
763.2998	0.8058	785.7000	0.9656
747.5000	0.7373	664.0000	0.8467
635.7998	0.6454	790.0000	0.9736
1263.0999	1.4755	513.2000	0.3631
987.0999	1.0228	1068.2998	1.2913
754.0999	0.9719	729.2000	0.6888
1119.2998	1.2125	545.2000	0.4929
683.0999	0.5515	695.8999	1.0859
743.0000	0.9086	1043.3999	1.0897
781.2998	0.8292	552.2000	0.5066
768.2998	0.9325	659.2000	0.9111
1282.2998	1.2384	767.7998	0.6995
609.2000	0.4810	681.7998	1.0879
725.3999	0.8956	925.7000	0.8321
523.8999	0.5425	704.2998	0.7786
562.2000	0.4558	694.5999	0.8220
934.2000	1.1240	1699.7000	1.9828
971.5000	1.3375	734.7998	0.9847
549.8999	0.5234	688.8999	0.8782
1369.2000	2.0692	849.7000	1.1444
558.0000	0.4664	551.2000	0.5754
743.0000	0.7145	1139.5000	1.5717
1042.5999	1.3151	941.7000	0.9080
862.5000	0.9340	987.3999	1.2192
1085.2998	1.1666	728.3999	0.8493
1248.7998	1.7349	881.0000	1.2301
729.2998	0.9287	1062.0000	0.8504
1085.8999	1.2003	749.8999	1.1255
1136.5999	1.1074	579.0000	0.7533
998.2000	1.1210	625.0000	0.8003
842.5000	0.8414	713.0000	0.9448
625.5000	0.6790		
967.5999	1.2432		
1022.7000	1.4037		
1006.7000	1.3606		
596.7000	0.6456		
1000.5999	1.3256		
660.0999	0.8086		
587.2998	0.7184		
1051.5000	1.2500		
658.8999	0.8999		
711.3999	0.8988		
677.5999	1.0285		
1231.5000	1.4286		
1004.3999	1.335		
595.3999	0.4473		
1255.7000	1.8550		
800.7000	1.1665		

TABLE A.3(Continued)

GAMMA	LAMBDA
1076.2000	1.2229
1277.7998	1.8438
1109.8999	1.3950
802.2998	0.9201
834.0999	1.2543
989.0999	1.0521
1392.0000	1.9634
1038.2998	1.3092
1250.7000	1.4110
829.3999	0.9717
728.2000	0.8217
807.3999	1.1283
1235.5999	1.2762
466.2998	0.4801
1047.2000	1.2111
1119.7998	1.4488
1199.2000	1.6194
725.8999	0.8500
652.7998	0.7714
868.5999	1.3025
723.8999	0.7434
734.7000	0.7315
681.2000	0.8583
1006.8999	1.3533
939.2000	0.9735

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RADC plans and conducts research, exploratory and advanced development programs in command, control, and communications (C^3) activities, and in the C^3 areas of information sciences and intelligence. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.